Measuring Substitution and Complementarity among Offers in Menu Based Choice Experiments

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Abstract

Choice experiments designed to extend beyond the classic application of choice among perfect substitutes have become popular in marketing research. In these experiments, often referred to as menu based choice, respondents face choice sets that may comprise substitutes, complements, and offers that provide utility independently, or any mixture of these three types. The inferential challenge posed by data from such experiments is in the calibration of utility functions that accommodate a mix of substitutes, complements, and independent offers. Moreover, while a prior understanding of the product categories under study may, for example, suggest that two offers in a set are essentially perfect substitutes, this may not be true for all respondents. To address these challenges, we combine Besag's (1972, 1974) autologistic choice model with a flexible hierarchical prior structure. We explain from first principles how the autologistic choice model improves on the multivariate probit model, and on models that include cross-price effects in the utility function. We develop Bayesian inference for the autologistic choice model, including its intractable normalizing constant and find empirical support for our model in a menu based conjoint experiment investigating demand for game consoles and accessories. We illustrate implications for optimal pricing.

Keywords: *menu based choice, choice modeling, autologistic choice model, hierarchical Bayes*

1 Introduction

Menu based choice experiments (MBCEs) that extend beyond the case of choice among perfect substitutes as in choice based conjoint (CBC) have become popular recently (Liechty et al., 2001; Orme, 2010). In analogy to the choice of a starter, a main course and a desert from a restaurant menu, MBCEs accommodate any combination of substitutes, complements and independent offers in the choice sets, i.e., the menus presented to respondents. As a consequence, the utility maximizing choice from such a choice set may be a combination of the individual offers available in the choice set. The prototypical MBCE presents each respondent with multiple menus to choose from, varying the prices or the availability of individual offers in the menus (Orme, 2010). The resulting data is used to calibrate choice models designed to rationalize the choice of the various combinations of individual offers from the menus. In turn, menu optimization then proceeds based on the calibrated choice model. Typical optimization problems involve determining profit maximizing prices of individual offers in a menu and more generally which offers to include. A critical input to these optimization problems are measures of demand interdependencies between offers in a menu.

Current modeling approaches account for demand interdependencies between offers in a menu using correlated errors (Liechty et al., 2001) or by including selected cross-price effects (Orme, 2010). A different approach that has not seen applications to MBCEs is the autologistic choice model (ALCM) developed by Besag (1972; 1974) and introduced into marketing by Russell and Petersen (2000).

We show, based on first principles, that the ALCM has important advantages over extant models applied to MBCEs. Specifically, we show that correlated errors as well as cross-price effects only provide a limited representation of complementarity and substitution between offers in a menu, with counterintuitive implications for optimal actions. We then develop Bayesian inference for a hierarchical version of the ALCM including a prior that supports strong utility dependencies, e.g., essentially perfect substitution among a subset of offers, but allows for individual level departures.

On the computational side we develop a method to construct good proposal densities based on approximate data augmentation that nevertheless maintains the detailed balance conditions. The method facilitates Metropolis-Hastings sampling in situations where large step sizes are required by construction of the model. We adopt the recently proposed exchange algorithm (Möller et al., 2006; Murray et al., 2006) to circumvent the evaluation of the normalizing constant in the ALCM's likelihood that is computationally intractable in larger choice sets. We thus avoid using the common pseudo-likelihood that turns out to be biased in our setting.

We apply our model to data from an MBCE designed to study demand for game consoles and accessories by GfK. We find empirical evidence for the relevance of utility dependencies because of substitution and complementarity in our data, demonstrate superior predictive performance relative to extant models, and illustrate implications for optimal pricing of offers in a menu.

In section 2 we review the ALCM and contrast it to the multivariate probit model (Liechty et al., 2001) and to including cross-price effects in independent binomial logit models (Orme, 2010). In section 3 we develop our hierarchical prior for the ALCM. Section 4 presents the data along with estimation results. Section 5 discusses pricing implications. We conclude with a summary and an outlook.

2 Models for Menu Based Choice

2.1 The Autologistic Choice Model

2.1.1 Definition and Characterization

Consider a menu containing $k = 1, \ldots, K$ offers. Restricting attention to binary choice of individual offers as common in MBCEs, there are $j = 1, \ldots, J$ possible choice outcomes where $J = 2^K$. This includes choosing nothing, i.e., the outside good from the menu. The ALCM puts a distribution on these choices.

One way to derive the model simply assumes that the choice among the J different combinations of offers is multinomial logit:

$$P(j) = \frac{exp(f(j))}{\sum_{h=1}^{J} exp(f(h))}, \tag{1}$$

where $f(j)$ is an essentially arbitrary characterization of the indirect utility associated with combination j.

An alternative derivation is by Besag (1972) who originally proposed the ALCM for the analysis of multivariate binary spatial data. His derivation is based on a causal argument about the nature of interactions between sites on a spatial array. The argument states that the conditional probability of an event at a focal site in the array only depends on observable states of directly connected neighboring sites, i.e., on whether an event has occurred at these sites or not, and possibly on non-spatial characteristics of the focal site.

In our context, the array is the menu a respondent chooses from, events on the array are choices of individual offers from the menu, and the connections between offers are due to complementarity or substitution.

Besag shows that this causal argument alone, together with the requirement that a proper joint distribution of events exists, implies (i) additive separability of $f(j)$, (ii) that conditional probabilities are in the form of logistic models, and that (iii) the joint probability of a particular configuration of events is in multinomial logit form. Implication (i) gives rise to indirect utility functions in the form of Equation 2.

$$f(j) = \sum_{k \in j} \beta_k + \theta(j), \quad f(j \equiv k) = \beta_k \tag{2}$$

Here β_k is a parameter reflecting the attractiveness of, or utility from offer k, and $\theta(j)$ measures the differential utility from the particular combination j of offers in the menu. Economic reasons for this differential utility are utility dependencies among the offers in j due to complementarity and substitution. Note that Equation 1 reduces to the familiar multinomial logit model when all $\theta(j)$ approach negative infinity.

To further exemplify the argument, define j' as that combination of offers that excludes offer k, but is otherwise equal to j, i.e., $j' \equiv j \setminus k$. From the ratio of the probability of j, i.e., the joint probability of choosing combination j including k, to the marginal probability of j' with respect to k, we see that the conditional probability of choosing k is in the form of a binomial logit model (Equation 3).

$$P(k|j' \equiv j \setminus k) = \frac{exp(f(j))}{exp(f(j')) + exp(f(j))} = \frac{exp\left(\beta_k + \theta(j) - \theta(j')\right)}{1 + exp\left(\beta_k + \theta(j) - \theta(j')\right)} \tag{3}$$

Equation 3 can be interpreted as the probability of choosing offer k presented after someone made choice j' from a menu that did not feature offer k. Remarkably, this conditional probability is independent of the attractiveness, and the substitutive or complementary nature of rejected offers in this menu, and independent of the offer specific attractiveness of conditioning arguments, i.e., independent of $\sum_{l \in j'} \beta_l$ given choice j', reflecting Besag's (1972) original causal argument.

In Equation 3, the direct effect from choice j' on the conditional choice probability of k is $(\theta(j) - \theta(j'))$. If offers in j' neither enhance nor subtract from the utility of k and vice versa, i.e., if there is no utility connection between k and offers in j', then $\theta(j) = \theta(j')$ and $P(k|j') = P(k|\neg j') = P(k)$. If there is a connection between offers in j' and k, and offers in j' collectively substitute k, then $\theta(j) < \theta(j')$ and thus $\theta(j) - \theta(j') < 0$. As an observable consequence, the probability of choosing k given choice j' is smaller than the probability of choosing k in the absence of j'. Finally, if offers in j' complement k, then $\theta(j) > \theta(j')$ and thus $\theta(j) - \theta(j') > 0$ such that $P(k|j') > P(k|\neg j')$.

Closely related, the sign of $(\theta(j) - \theta(j'))$ determines the sign of the change in the marginal probability of choosing k, defined in Equation 4, when changing the attractiveness of offers in j'. To obtain the marginal probability in Equation 4, we sum the probabilities of choosing only k and that of choosing k together with j', i.e., choosing j from a menu that contains k and j'.

$$P(k) = \frac{exp(\beta_k) + exp(\sum_{l\in j} \beta_l + \theta(j))}{1 + exp(\beta_k) + exp(\sum_{l\in j'} \beta_l + \theta(j')) + exp(\sum_{l\in j} \beta_l + \theta(j))} \tag{4}$$

The cross-derivative of this marginal probability with respect to the attractiveness of offers in j' in Equation 5 will be positive when $(\theta(j) - \theta(j')) > 0$, i.e., when j' and k complement each other. In this case, increasing the attractiveness of offers in j' will increase the marginal probability of choosing k, and vice versa when $(\theta(j) - \theta(j')) < 0$, i.e., when j' and k substitute each other.[1]

$$\frac{dP(k)}{d\sum_{l\in j'} \beta_l} = \frac{exp(\sum_{l\in j} \beta_l + \theta(j)) - exp(\sum_{l\in j'} \beta_l + \beta_k + \theta(j'))}{(1 + exp(\beta_k) + exp(\sum_{l\in j'} \beta_l + \theta(j')) + exp(\sum_{l\in j} \beta_l + \theta(j)))^2} \tag{5}$$

As we will show further below, the multivariate probit model that was proposed as an alternative for the analysis of MBCEs (Liechty et al., 2001) fails at capturing the cross-derivates implied by substitution or complementarty between offers.

2.1.2 Specification

In a fixed menu of K offers the corresponding $J = 2^K$ choice probabilities—assuming we have enough data to compute them directly—just identify the parameters in Equation 2 subject to a normalization required because $\sum_{j=1}^{J} P(j) = 1$. As customary in choice models, we normalize the utility of the outside good to zero, i.e., set $f(j \equiv \emptyset) = 0$. Therefore, the model so far does not impose any constraints on the choice probabilities from a given menu. However, the model does constrain probabilities across choices from various subsets of the K total offers because of the IIA-assumption in Equation 1.

Nevertheless, in but the smallest menus, an attempt to calibrate all $j = 1, \ldots, (J-1)$ model parameters from observed choices is hopeless because the number of observations is likely smaller than the number of parameters. For example, in a menu with $K = 20$

[1]Note that $\sum_{l\in j} \beta_l = \sum_{l\in j'} \beta_l + \beta_k$ in the numerator of Equation 5.

offers, we need data to estimate $2^{20} = 1,048,576$ choice probabilities to compute the corresponding $2^{20} - 1$ parameters.

For additional structure and substantially more parsimony, we therefore define every $\theta(j)$ as the sum of bivariate relationships among the offers that constitute choice j:

$$\theta(j) = \sum_{k=1}^{K} \sum_{k'=k+1}^{K} \theta_{k,k'} I(k \in j) I(k' \in j). \tag{6}$$

Here $I(arg)$ evaluates to one if arg is true and else to zero. This constraint reduces the number of parameters to estimate from $2^K - 1$ to $(K^2 + K)/2$. The resulting model is still extremely flexible in that every offer potentially interacts with every other offer in the menu, but rules out higher order interactions. An example for a higher order interaction are three individual offers that only become attractive when chosen together.

Under the constraint in Equation 6 it is useful to define the matrix Θ as

$$\Theta = \begin{pmatrix} 0 & \theta_{2,1} & \dots & \theta_{K,1} \\ \theta_{2,1} & 0 & \dots & \vdots \\ \vdots & \dots & \ddots & \theta_{K,K-1} \\ \theta_{K,1} & \dots & \theta_{K,K-1} & 0 \end{pmatrix} \tag{7}$$

and $\mathbf{y}(j) = [y_1(j), \dots, y_k(j), \dots, y_K(j)]'$ where $y_k(j) = 1$ if $k \in j$ else $y_k(j) = 0$. Then the double sum in Equation 6 can be more compactly expressed in matrix form as

$$\sum_{k=1}^{K} \sum_{k'=k+1}^{K} \theta_{k,k'} I(k \in j) I(k' \in j) = \mathbf{y}(j)' ltr(\Theta) \mathbf{y}(j) \tag{8}$$

where ltr is short for lower-triangular.

Many MBCE's include utility shifters, such as e.g., prices of individual offers that vary across menus as part of an experimental design. We follow the standard practice of including price as 'linear attribute' in the (indirect) utility of individual offers:

$$\beta_k = \beta_{k0} + \beta_{price} p_k, \tag{9}$$

Equation 9 implies a quasilinear utility specification for choices $j = 1, \dots, J$ from a menu (see Equation 2) from which income drops out. Therefore, the utility interactions $\theta_{k,k'}$ should be interpreted as interactions in direct utility. For example, a negative element $\theta_{k,k'}$ is negative because alternative k makes alternative k' redundant, independent of the income consumed by purchasing k.

2.2 The Multivariate Probit Model

Liechty et al. (2001) proposed the multivariate probit model (MvP) as basic model for data from MBCEs with the understanding that error correlations capture complementary and substitutive relationships between offers in the menu. However, while error correlations in the MvP can certainly *reflect* dependencies from complementarity or substitution, the MvP fails at *generating* important implications from the economics of substitution and complementarity. Most notably, all cross derivatives in the MvP, and thus all cross-price elasticities are zero by definition of the model if the only connection between offer-specific utilities is through correlated errors.

This property of the MvP is best illustrated using an example. Consider a menu consisting of the two alternatives A and B with attractiveness or utility a and b, and define the corresponding random utilities as z_A and z_B. Then the marginal probability of choosing B from this two item menu is defined as $P(z_B > 0)$. Starting from the joint distribution $p(z_A, z_B)$, this probability corresponds to the following double integral:

$$\begin{aligned} P(B) &= \int_0^\infty \int_{-\infty}^\infty N\left(\begin{bmatrix} z_A \\ z_B \end{bmatrix} \mid \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} 1 & \rho_{AB} \\ \rho_{AB} & 1 \end{bmatrix}\right) dz_A dz_B \\ &= \int_0^\infty N\left(z_B|b,1\right) dz_B \end{aligned} \tag{10}$$

where ρ_{AB} is the correlation between random utilities z_A and z_B. As obvious from the last line of Equation 10, $dP(B)/da = 0$, i.e., the marginal share of B is independent of the attractiveness of A for all latent utility correlations ρ_{AB}. This implication of the MvP is inconsistent with the economics of substitution and complementarity that imply decreasing (increasing) demand for the substitute (complement) B as the attractiveness of A increases. It follows that in applications where a prior understanding of the offers in a menu suggest that cross-derivatives are likely to be non-zero, e.g., that increasing the price of offer k will shift demand from k to k', an account that is based on correlated errors only is inadequate (cf. Equation 5).

A related question is how to empirically distinguish between dependencies that arise from utility correlations as in the MvP and utility interactions as in the ALCM. We noted earlier that repeated choices from a fixed menu are sufficient to just identify the ALCM parameters in Equation 2. In a menu comprised of only the two offers A and B, the data yield four probabilities $P(A)$, $P(B)$, $P(A, B)$, and $P(\emptyset)$ that can identify three linearly independent parameters. Thus, before invoking special cases such e.g., perfect substitution between two individually attractive offers, both the ALCM and the MvP are just identified and thus empirically indistinguishable in this example.

However, exogenous variation in the base-line probability of A or B will distinguish between the ALCM and the MvP. Consider for example choices from three different 'menus' consisting of $\{A\}$, $\{B\}$, and $\{A, B\}$, respectively and note that the probability of choosing, e.g., B from the menu consisting of only $\{A\}$ is exogenously equal to zero. According to the MvP, the marginal probability of choosing, e.g., A is independent of the menu context, i.e., $P(A|\{A\}) = P(A|\{A, B\})$, see Equation 10. In contrast, the ALCM will predict $P(A|\{A\}) \neq P(A|\{A, B\})$, unless there is no utility interaction between offers A and B, i.e., $\theta_{AB} = 0$ (see Equations 4 and 5). The same argument holds if there are utility shifters that vary the attractiveness of individual offers in a menu exogenously.

Another related question is if correlations between latent utilities as in the MvP and utility interactions as in the ALCM can be jointly identified. We will defer a thorough answer to this question to future research. Gentzkow (2007) who estimates both utility interactions and correlations for the two-alternative case of this model offers some discussion. In our application to MBCEs, substitution and complementarity together with persistent heterogeneity are the first order concerns. We will leave the discussion of direct utility interactions versus utility correlations at the level of model comparisons.

2.3 Cross-Price Effects

The current industry standard to handle demand dependencies among offers in MBCEs is to include selected cross-price effects into the offers' utility functions (Orme, 2010). The random utility of offer k is specified as

$$u_k = \beta_{k0} + \sum_{k' \in (1,\ldots,K)} \beta_{price,k,k'} p_{k'} + \epsilon_k, \; \epsilon_k \sim Logistic \tag{11}$$

resulting in independent binomial logit models for the K offers in a menu. The intuition is that substitution leads to positive and complentarity to negative cross-price effects.

However, this approach suffers from two conceptual problems. First, the assumption that offer k's utility depends on the price of k' independent of whether k' is chosen or not contradicts basic utility theory. As a practical consequence, optimization over prices given parameters is bound to yield solutions that suggest to maximize demand for a high-margin offer by setting the price for substitutes to infinity, for common functional forms of the cross-price effects. Second, and in contrast to aggregate demand models, substitution and complementarity exert their influence on choices even in the absence of price variation. Regardless of the estimated cross-price effects, the model in Equation 11 fails at *generating* substitution between offers whenever two substitutes are priced such that their indirect utilities are equal. Last but not least, crossprice effects may even fail to correctly *reflect* the sign of utility interactions, depending on the amount of price variation in the data (see Appendix D for a numerical illustration).

We conclude that the ALCM has important theoretical advantages over the MvP and independent binomial logit models that include cross-price effects (IndepCPE) as a basic model for MBCEs. Next we develop Bayesian inference for this model including a parsimonious prior for the utility interaction parameters in the ALCM.

3 Bayesian Inference

The ALCM described in the previous Section 2.1 implies the following individual i ($i = 1, \ldots, N$) specific indirect utility function for choice j consisting of some combination of offers from menu t ($t = 1, \ldots, T$) with a total of K offers:

$$f_{i,t}(j) = \sum_{k=1}^{K} y_k(j)(\beta_{k,i} + \beta_{price,i} p_{k,t}) + \mathbf{y}(j)' ltr(\boldsymbol{\Theta}_i)\mathbf{y}(j) \tag{12}$$

Inference for the parameters in Equation 12 is challenging, and in particular in the context of MBCEs. First, the normalizing constant of the likelihood in Equation 1 has $J = 2^K$ terms and becomes prohibitively expensive to compute in larger menus. We propose a solution to this problem enabling likelihood based Bayesian inference based on the recently developed exchange algorithm (Möller et al., 2006; Murray et al., 2006). Second, in MBCEs each of N respondents typically only make choices from a small set of different menus, i.e., T is small and $N >> T$. Therefore it is natural to attempt inference using a hierarchical model that pools information across respondents. Next we briefly describe the exchange algorithm and then introduce our hierarchical prior.

3.1 The Exchange Algorithm

Define $\mathbf{Y}_i = \left(\mathbf{y}'_{i,1}, \ldots, \mathbf{y}'_{i,T}\right)$ as the matrix of choices by individual i from T different menus, where $y_{i,t,k} = 1$ if individual i chooses offer k from menu t and else zero such that each $\mathbf{y}_{i,t} = \mathbf{y}(j)$ for some $j \in (1, \ldots, J)$, and $\boldsymbol{\vartheta}_i = (\boldsymbol{\beta}_i, \boldsymbol{\Theta}_i)$.

Denote by $p(\boldsymbol{\vartheta}_i|\mathbf{Y}_i)$ the *normalized* posterior, $P(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i)$ the normalized likelihood defined by Equations 1 and 12 with normalization constant $nc_t(\boldsymbol{\vartheta}_i)$ (the denominator in Equation 1) at the t-th observation, $\ell^*(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i)$ the *non*-normalized likelihood (the numerator in Equation 1), $p(\boldsymbol{\vartheta}_i)$ as the prior and $q(\boldsymbol{\vartheta}_i)$ as the proposal density. Then, the exchange algorithm uses the following acceptance probability:

$$\alpha\left(\boldsymbol{\vartheta}_i, \mathbf{Y}_i \rightarrow \boldsymbol{\vartheta}_i^c, \mathbf{Y}_i^c\right) = min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i^c\right) q\left(\boldsymbol{\vartheta}_i\right)}{p\left(\boldsymbol{\vartheta}_i\right) q\left(\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}\right) \tag{13}$$

where $\mathbf{y}_{i,t}^c \sim \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)}$ is obtained by 'Gibbsing through' the corresponding set of conditional distributions defined in Equation 3, or from perfect sampling, and *without* evaluating $nc_t\left(\boldsymbol{\vartheta}_i^c\right)$ (see Appendix A).

The key insight behind the exchange algorithm is that by expanding the MCMC-state space from $\boldsymbol{\vartheta}_i$ to $(\boldsymbol{\vartheta}_i, \mathbf{Y}_i)$, the normalization constants $nc_1(\boldsymbol{\vartheta}_i), \ldots, nc_T(\boldsymbol{\vartheta}_i)$ and $nc_1\left(\boldsymbol{\vartheta}_i^c\right), \ldots, nc_T\left(\boldsymbol{\vartheta}_i^c\right)$ cancel from the MH-acceptance ratio. We obtain the 'traditional', marginal MH-acceptance ratio by integrating out $\mathbf{Y}_i^c$:

$$\begin{aligned}
\alpha\left(\boldsymbol{\vartheta}_i \rightarrow \boldsymbol{\vartheta}_i^c\right) &= min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i^c\right) q\left(\boldsymbol{\vartheta}_i\right)}{p\left(\boldsymbol{\vartheta}_i\right) q\left(\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \int \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)} d\mu(\mathbf{y}_{i,t}^c)\right) \\
&= min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i^c\right) q\left(\boldsymbol{\vartheta}_i\right)}{p\left(\boldsymbol{\vartheta}_i\right) q\left(\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)} \frac{nc_t\left(\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)}\right) \\
&= min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i^c\right) q\left(\boldsymbol{\vartheta}_i\right)}{p\left(\boldsymbol{\vartheta}_i\right) q\left(\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{P\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{P\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}\right)
\end{aligned} \tag{14}$$

While the relation between Equations 13 and 14 serves as an intuitive justification of the exchange algorithm, we prove its detailed balance with respect to the normalized posterior $p(\boldsymbol{\vartheta}_i|\mathbf{Y}_i)$ in Appendix B.

Some may view the pseudo-likelihood (PL) proposed by Besag (1972) as a simple, viable alternative. The PL conveniently approximates the joint likelihood function by the product of full conditional distributions:

$$PL(\boldsymbol{\vartheta}_i) = \prod_{t=1}^{T} \prod_{k=1}^{K} P\left(y_{i,t,k}|\boldsymbol{\vartheta}_i, \mathbf{y}_{i,t,-k}\right)^{y_{i,t,k}} \left(1 - P\left(y_{i,t,k}|\boldsymbol{\vartheta}_i, \mathbf{y}_{i,t,-k}\right)\right)^{(1-y_{i,t,k})} \tag{15}$$

where $P(y_{i,t,k}|\boldsymbol{\vartheta}_i, \mathbf{y}_{i,t,-k})$ is respondent i's conditional choice probability of offer k in menu t defined in Equation 3, and $\mathbf{y}_{i,t,-k}$ is short for $\mathbf{y}_{i,t} \setminus y_{i,t,k}$. The conditional choice probabilities are in the form of binomial logits and their product is easily computed, even for extremely large menus. For the special case of $\boldsymbol{\Theta} = \mathbf{0}$, but only for this special case, we have that the product of conditional choice probabilities in Equation 15 yields Equation 1.

In the presence of utility interactions, i.e., with $\boldsymbol{\Theta} \neq \mathbf{0}$, the PL is only large-sample consistent (e.g., Särkkä, 1996; Zhao and Joe, 2005). Other authors report that the difference between PL and likelihood based inference is only in efficiency (Gong and Samaniego, 1981). However, the situation is somewhat more complicated in the context of a hierarchical model. The successive conditioning on $\mathbf{y}_{i,t,-k}$ inherent to the PL violates the assumption of independence between the distribution of conditioning arguments and the distribution of parameters across respondents. The resulting bias can be substantial (e.g., Manchanda et al., 2004; Liu et al., 2007).

3.2 Hierarchical Prior

The standard hierarchical prior formulation, i.e., a multivariate normal prior coupled with the likelihood implied by Equations 1 and 12 is well suited for parameters ($\beta_{i,1}, \ldots,$ $\beta_{i,k}, \ldots,$ $\beta_{i,K}$, $\beta_{i,price}$) that characterize the attractiveness of individual offers. It is less useful for inference about the utility-interaction parameters in $\boldsymbol{\Theta}_i$. The reason is heterogeneity in what respondents perceive to be perfect substitutes.

Recall that utility interactions $\theta_{i,k,k'}$ in $\boldsymbol{\Theta}_i$ measure interactions in direct utility. For example, if respondent i receives no additional direct utility at all from offer k' once he chooses offer k and vice-versa, the corresponding parameter $\theta_{i,k,k'}$ is negative infinity for this respondent. However, if another respondent perceives horizontal differences between offers k and k' such that choosing k and k' together becomes a possibility, $\theta_{k,k'}$ for this respondent is 'infinitely' larger than for respondent i.

The standard approach to accommodating such extreme forms of heterogeneity is to use a hierarchical prior defined as a discrete mixture of distributions. In the following, we develop a hierarchical prior for $\boldsymbol{\Theta}_i$ based on this idea. However, we pool information across individual elements $\theta_{i,k,k'}$ based on a prior understanding of the nature of the bi-variate interactions. Pooling across individual elements $\theta_{i,k,k'}$ is necessary because of the limited amount of likelihood information to inform individual parameters $\theta_{i,k,k'}$ in $\boldsymbol{\Theta}_i$. The individual level likelihood of a respondent's choices across a (short) sequence of menus is maximized by setting all $\theta_{i,k,k'}$ that correspond to offers that were never chosen together to negative infinity, for example. The implication that all these pairs are perfect substitutes to this respondent is likely to be wrong.

Our hierarchical prior classifies each element $\theta_{i,k,k'}$ for all $k = 1, \dots, K,\ k' = k + 1, \dots, K,\ i = 1, \dots, N$ into one of three different 'classes' based on an ordinal-probit regression. Equation 16 shows the 'latent' linear equation underlying the ordinal-probit and Figure 1 depicts how the latent dependent variable $z_{i,k,k'}$ in this regression maps into the distinct classes denoted S, I, and C based on fixed truncation points at zero and one. The class labels S, I, and C are intentionally chosen to relate to *s*ubstitution, *i*ndependence, and *c*omplemenartiy. We will revisit this point below.

$$z_{i,k,k'} = \mathbf{w}'_{k,k'}\boldsymbol{\delta} + \epsilon_{i,k,k'}, \quad \epsilon_{i,k,k'} \sim N(0, \sigma^2_\epsilon) \tag{16}$$

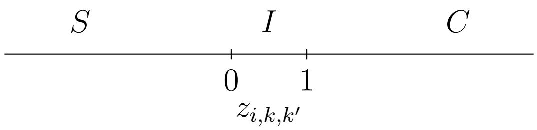

Figure 1: Mapping from $\mathbf{w}'_{k,k'}\boldsymbol{\delta}$ to θ-Classes

Prior knowledge about $\theta_{i,k,k'}$ is encoded in the design vector $\mathbf{w}_{k,k'}$. If a particular bivariate interaction $\theta_{i,k,k'}$ is a priori expected to be similar to that between k and k'', i.e., $\theta_{i,k,k'} \simeq \theta_{i,k,k''}$ for all $i = 1, \dots, N$, for example, then $\mathbf{w}_{k,k'}$ is equal to $\mathbf{w}_{k,k''}$. However, the classification implied by the ordinal-probit model is probabilistic for all $\sigma^2_\epsilon > 0$ and $-\infty < \mathbf{w}'_{k,k'}\boldsymbol{\delta} < \infty$, such that the likelihood information in a respondent's choices may result in a posterior classification of a particular element $\theta_{i,k,k'}$ different from its (most likely) prior classification.

Conditional on classifications of individual $\theta_{k,k',i}$, we specify hierarchical normal, inverse Gamma (N-IG) priors for the distribution of θ-elements in each class:

$$\begin{aligned} \bar{\theta} &\sim N(0, a^{-1}) \\ V_\theta &\sim IG(\nu, s) \end{aligned} \tag{17}$$

To compensate for the limited individual level likelihood information and to regularize our inference problem, we assume that θ-elements, given classifications, are (relatively) tightly distributed around some location. For the S- and the C-class we essentially fix

V_θ equal to one but use a diffuse prior for $\bar{\theta}$. For the I-class $\bar{\theta}$ is fixed at zero and V_θ constrained to be practically zero.

To illustrate further, consider the following matrix of design vectors $\mathbf{w'}_{k,k'}$:

$$\mathbf{W} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

The columns correspond to coefficients $\boldsymbol{\delta}$ in Equation 16, where the first coefficient is a constant and the following coefficients measure departures from that constant. For example, $\boldsymbol{\delta} = (.5, -5, 6)'$ implies that the θ-parameter corresponding to line one in the matrix above, and all other θ-parameters with the same design vector, a priori most likely connect two $i$ndependent offers such that they should be zero (see Figure 1). For line two and three in the matrix above, these $\boldsymbol{\delta}$-values imply that the corresponding θ-parameters are similar, but different from zero. In principle, the likelihood will determine if they are smaller ($s$ubstitution) or larger than zero ($c$omplementarity). However, depending on the prior knowledge about thus grouped θ-elements, the prior expectation in one of the two directions will be strong.

For lines four and five in the matrix above these $\boldsymbol{\delta}$-values again imply that the corresponding θ-parameters are similar, different from zero, and likely different from the θ-parameters corresponding to lines two and three. Again, the likelihood will determine if they are smaller ($s$ubstitution) or larger than zero ($c$omplementarity). And again, depending on the prior knowledge about thus grouped θ-elements, the prior expectation in one of the two directions will be strong. We envision that in most applications, prior information will be rich enough to both establish expected similarities among θ-parameters as well as an expectation about the directional departure from zero for at least some of the resulting groupings.

More formally, our hierarchical ordinal-probit model implies the following prior classification probabilities for θ-parameters depending on the corresponding $\mathbf{w}$-vectors, and parameters $\boldsymbol{\delta}$ and σ_ϵ^2 in the hierarchical classification prior:

$$P(\theta_{i,k,k'} \in S | \mathbf{w'}_{k,k'}\boldsymbol{\delta}, \sigma_\epsilon^2) = \int_{-\infty}^{0} N(z_{k,k'} | \mathbf{w'}_{k,k'}\boldsymbol{\delta}, \sigma_\epsilon^2) dz \tag{18}$$

$$P(\theta_{i,k,k'} \in I|\mathbf{w'}_{k,k'}\boldsymbol{\delta}, \sigma^2_\epsilon) = \int_0^1 N(z_{k,k'}|\mathbf{w'}_{k,k'}\boldsymbol{\delta}, \sigma^2_\epsilon)dz \tag{19}$$

$$P(\theta_{i,k,k'} \in C|\mathbf{w'}_{k,k'}\boldsymbol{\delta}, \sigma^2_\epsilon) = \int_1^{+\infty} N(z_{k,k'}|\mathbf{w'}_{k,k'}\boldsymbol{\delta}, \sigma^2_\epsilon)dz \tag{20}$$

The hierarchical prior on individual elements $\theta_{i,k,k'}$ established in Equations 16 through 20 has the following desirable properties: i) it allows a collection of θ-parameters believed to be similar a priori to take extreme values such that they can e.g., reflect essentially perfect substitution among a collection of offers in a menu, because ii) it does not impose these extreme values for all respondents, as the posterior classification of an element $\theta_{i,k,k'}$ may depart from its prior classification. Moreover, the posterior of the parameters in the hierarchical classification prior, i.e., $\boldsymbol{\delta}$ and σ^2_ϵ in Equations 16 and 18 through 20 will reveal if the prior grouping of θ-parameters is supported by the data or not. For example, $\delta \neq 0$ coupled with small σ^2_ϵ points to agreement between prior and posterior classifications. In summary, our adaptive prior anticipates concentrations of θ-parameters around some negative value ($\bar{\theta}^{(S)}$), at zero ($\bar{\theta}^{(I)} \equiv 0$) and around some positive value ($\bar{\theta}^{(C)}$).

A theoretical drawback of the prior developed above is that it allows offers to be classified as substitutes a posteriori even if the prior expectation is that they are complements and vice-versa. In many applications such reclassifications will be regarded as a result of overfitting the data. In this case it is useful to restrict the applicability of Equations 18 to 20 to a subset of θ-parameters, and to apply the following prior classification rules to θ-parameters that are believed to connect substitutes, a priori:

$$P(\theta_{i,k,k'} \in S|\mathbf{w'}_{k,k'}\boldsymbol{\delta}^S, 1) = \int_{-\infty}^0 N(z_{k,k'}|\mathbf{w'}_{k,k'}\boldsymbol{\delta}^S, 1)dz \tag{21}$$

$$P(\theta_{i,k,k'} \in I|\mathbf{w'}_{k,k'}\boldsymbol{\delta}^S, 1) = 1 - p(\theta_{i,k,k'} \in s|\mathbf{w'}_{k,k'}\boldsymbol{\delta}^S, 1) \tag{22}$$

Equations 21 and 22 constrain the departure from the prior hypothesis of substitution between two offers to independence and thus avoid the reclassification of prior substitutes into posterior complements deterministically. Obviously, the same approach can be used to avoid the reclassification of prior complements into posterior substitutes deterministically as well.

In our illustrative case study, we will refer to this restricted prior as 'ALCMhet' and compare it to an ALCM where $\boldsymbol{\Theta}_i \equiv \boldsymbol{\Theta}$, i.e., all utility interactions are assumed to be homogeneous. We will refer to this model as 'ALCMhom'.

We defer the details of Bayesian inference based on the hierarchical priors discussed here to technical Appendix C. In this Appendix we develop approximate data augmentation for this model while maintaining the detailed balance conditions to address the problem of the large Metropolis-Hastings step-sizes that result from attempting to reclassify individual interaction parameters $\theta_{i,k,k'}$.

4 Illustrative Case Study

4.1 Data

We illustrate our method using data from an MBCE featuring game consoles and console accessories. The experiment was conducted in 2013 by GfK for strictly academic purposes, however, mimicking typical client demands. Respondents were recruited from an online panel and pre-screened for market-membership. If a prospective respondent had no console at home and was unlikely to purchase one within the next year, the experiment terminated after the screening questions. Overall, 575 respondents participated in the MBCE who were 'in the market' according to the screening questions. Table 1 summarizes demographics of the sample used for estimation in our analysis. The median respondent in our sample is male, about 42 years old, lives in a three-person household, and has a high school degree. Overall, about 70% of respondents have at least one console among the set Xbox360, Xbox, PS3, PS2, Wii, or WiiU at home, whereby 41% own exactly one console, 19% two consoles, and 10% own more than two consoles. The remaining respondents either do not own game consoles (26%) or do not use any of the consoles they own any longer (4%). The average (median) console owner spends about 13 (6) hours a week playing video games, with a standard deviation of about 20.

Table 2 summarizes the distribution of game genres played by respondents conditional on the type of console owned. For example, 69% of respondents who own a PS3 console, use it for playing action games. Fitness games appear to be particularly popular among consumers who own a Nintendo console (Wii with 66% and WiiU with 62% compared to other consoles between 9% and 29%). On the other hand, Nintendo consoles are used less for playing adventure (Wii with 41% and WiiU with 33%) or racing games (Wii with 38% and WiiU with 29%). The differences between conditional game genre distributions in Table 2 are suggestive of horizontal differentiation between different consoles.

In our experiment, each individual choice menu was designed in the form of a webshop, where respondents could put products into an electronic shopping basket by clicking on

Table 1: Sample Demographics

Variable	Categories	Percentage
Age	$\leq$ 20	5.1
	21-40	41.5
	$\geq$ 41	53.4
Gender	female	42.8
	male	57.2
HH size	1 Person	15.4
	2 Persons	32.9
	3 Persons	24.7
	4 Persons	21
	$\geq$ 5 Persons	6
HH income (monthly, in Euros)	<1000	3.5
	1000-2000	15.7
	2000-3000	26.1
	3000-4000	23.6
	>4000	12.8
	missing	18.3
Education	primary/secondary	12.4
	high school	57.2
	college/university	30.4

Table 2: Game Genres

	Action	Adventure	Racing	Sports	Fitness	Strategy	Other
PS3	.69	.59	.60	.48	.18	.30	.00
PS2	.51	.54	.54	.43	.09	.22	.09
Xbox	.57	.52	.43	.38	.29	.43	.00
Xbox 360	.68	.60	.59	.51	.19	.30	.00
Wii	.33	.41	.38	.50	.66	.16	.03
WiiU	.52	.33	.29	.48	.62	.33	.05

them. The shopping basket always clearly displayed the total expenditure, i.e., the sum of the prices of individual offers in the shopping basket to respondents. Respondents could explicitly indicate that they would not even purchase a single offer from a particular menu using an additional no-choice option. Each respondent was exposed to twelve menus and clearly instructed to treat them independently. For each respondent we exlcuded two menus from estimation for predictive validation. Each menu comprised the same thirteen products, however at varying prices.

Products presented in each menu were from three different brands, i.e., Microsoft, Sony, and Nintendo. The design included the following game consoles and game console accessories: Xbox 360, Xbox One, PS3, PS4, Wii, Wii U, and Xbox Kinect, Xbox Wheel, PS Eye Cam, PS Move, PS Wheel, Wii Wheel, and Wii Motion. The technical specifications of game consoles were constant across all menus, and respondents could instantly retrieve the information on technical details in each menu by hovering the mouse over a particular offer. All consoles came with basic equipment, i.e., were usable without purchasing additional offers. The prices of individual offers varied exogenously across different menus.

In our data about 20% of respondents (122 out of 575) chose the outside option in every choice set. We exclude these respondents from our analyses as their data is void of information about relative preferences and substitution or complementarity among offers in the menus. Thus, we rely on a sample of 453 respondents and 4530 choices in total for estimation. Out of these 4530 choices, at least one item is chosen on 3403 occasions. 1669 choices contain at least two items from one menu, 613 choices at least three items, and finally 190 choices more than three items. Table 3 details marginal and bi-variate choice counts for the 13 products aggregated over respondents and repeated measurements. For example, PS4 was chosen 1214 times in total and PS4 and PS Wheel were chosen together 277 times, marginal with respect to other offer choices that may have accompanied this pair. Inspection of the table reveals higher dependence between game consoles and accessories of the same brand compared to other pairs of products. Moreover, it is apparent that different consoles are sometimes chosen together and therefore are not uniformly viewed as perfect substitutes. Respondents, who chose two or more consoles together at least once spend more time on gaming than those who chose at most one console from a menu with means (medians) of 16.9 (9) and 12.02 (5) hours per week, respectively.

Table 3: Frequencies of (Pairwise) Choices

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel	Wii Motion
Xbox 360	290												
Xbox One	12	345											
Xbox Kinect	67	62	222										
Xbox Wheel	83	75	55	243									
PS3	35	9	10	20	395								
PS4	29	81	24	39	30	1214							
PS Eye	12	20	31	24	69	215	402						
PS Move	13	34	28	41	65	305	91	481					
PS Wheel	16	27	13	48	67	277	112	158	432				
Wii	42	14	28	32	34	43	26	17	20	538			
Wii U	14	42	26	44	6	90	28	37	31	41	452		
Wii Wheel	21	30	39	54	15	37	32	37	39	116	134	433	
Wii Motion	31	35	35	48	25	64	40	55	46	238	201	193	634

We find that 29 respondents never chose a game console but only accessories from the 10 menus used for estimation. All of them report to already have at least one game console among the set Xbox360, Xbox, PS3, PS2, Wii, or WiiU at home. 127 respondents, at least once, only chose accessories from a menu. Of these, about 80% (101) report to have a game console at home. Among the 297 respondents that never chose accessories only from a menu, only about 68% (202) report to have a game console at home. We will revisit this feature of our data below, when we explain the set of models we compare.

4.2 Models

In addition to the various ALCM specifications: ALCM with homogeneous $\mathbf{\Theta}$ (ALCMhom), ALCM with heterogeneous $\mathbf{\Theta}$ with informative prior (ALCMhet), and a version of this model which includes inventory (ALCMhetInv) that we describe next, we include three benchmark models in our empirical model comparison: an independence model obtained from the ALCM by setting all utility interactions to zero (Indep), the multivariate probit model (MvP) and an independence model with selected cross-price effects (IndepCPE). We note that all models included in this comparison account for heterogeneity in preferences for individual offers and the price parameter using a standard unrestricted multivariate normal hierarchical prior. The models differ only in the way dependence between offers in a menu is accounted for.

ALCMhet uses the following two rules in its hierarchical prior specification: i) utility interactions, i.e., θ-parameters that connect two consoles are a priori similar to each other, and expected to be negative. As an exception a particular interaction $\theta_{k,k',i}$ may equal zero, but cannot be positive; ii) utility interactions that connect a console to accessories of the same brand are a priori similar and expected to be positive. As an exception a particular interaction $\theta_{k,k',i}$ may equal zero, but cannot be negative.

ALCMhetInv extends ALCMhet by additionally conditioning on whether at least one console among the set Xbox360, Xbox, PS3, PS2, Wii, or WiiU is currently owned. The technical implementation expands the choice vector $\mathbf{y}_{i,t} = [y_{1,t}, \ldots, y_{K,t}]'$ by one element to $\mathbf{y}_{i,t} = [y_{1,t}, \ldots, y_{K,t}, y_{i,inv}]'$, where $y_{i,inv}$ is respondent specific but takes the same value for all $t = 1, \ldots, T$ menus a respondent chooses from. It takes the value 1 if a respondent reported to own a console in the set Xbox360, Xbox, PS3, PS2, Wii, WiiU and 0 otherwise. Correspondingly, we expand the matrix $\mathbf{\Theta}_i$ for all $i = 1, \ldots, N$ by one row and one column. The elements $\theta_{i,K+1,k'}$ in this row ($\theta_{i,k',K+1}$ in this column) measure substitution and complementarity between the game console at home and the offers in the menus presented as part of our experiment. Elements $\theta_{i,K+1,k'}$ that connect the game consoles at home to consoles in a menu are a priori assumed to be similar and negative, in the same way as elements $\theta_{i,k,k'}$ that connect consoles offered in a menu to each other. We refrain from prior rule based (soft) classifications of $\theta_{i,K+1,k'}$ that connect the game console at home to accessories offered in a menu.

4.3 Results

Table 4 summarizes posterior means of alternative-specific constants and price-sensitivity at the hierarchical prior level. Posterior standard deviations, i.e., 'Bayesian standard errors' are reported in parentheses. The rank order of mean preferences across models is mainly consistent. For example, newer versions of the game consoles (Xbox One, PS4, and Wii U) deliver on average higher utility to respondents than older versions (Xbox 360, PS3, and Wii) and consoles deliver higher stand-alone utility than accessories. However, there exist a number of systematic differences between models.

Table 4: Price Sensitivity and Alternative Specific Constants - Hierarchical Prior Means

	ALCMhet	ALCMhet-Inv	ALCMhom	Indep	MvP	IndepCPE
Price	-0.34	-0.35	-0.30	-0.30	-0.13	-0.67
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.04)
Xbox 360	-0.20	2.38	-0.66	-2.05	-1.34	4.70
	(0.32)	(0.33)	(0.31)	(0.38)	(0.19)	(1.26)
Xbox One	3.67	6.42	2.43	0.63	0.37	8.88
	(0.47)	(0.45)	(0.42)	(0.50)	(0.26)	(1.10)
Xbox Kinect	-3.33	-2.55	-2.48	-3.39	-2.20	4.23
	(0.29)	(0.34)	(0.27)	(0.35)	(0.20)	(0.75)
Xbox Wheel	-4.39	-3.60	-4.56	-4.77	-2.78	5.33
	(0.34)	(0.30)	(0.32)	(0.45)	(0.18)	(0.86)
PS3	0.10	2.60	0.26	-1.24	-0.83	1.89
	(0.35)	(0.30)	(0.28)	(0.34)	(0.16)	(0.80)
PS4	5.13	7.88	4.08	3.60	1.77	12.69
	(0.49)	(0.60)	(0.45)	(0.54)	(0.23)	(1.03)
PS Eye	-3.54	-3.06	-3.08	-3.04	-1.95	5.81
	(0.25)	(0.24)	(0.20)	(0.23)	(0.13)	(0.73)
PS Move	-1.96	-1.60	-1.95	-1.88	-1.23	12.02
	(0.26)	(0.26)	(0.25)	(0.26)	(0.14)	(1.03)
PS Wheel	-4.20	-3.75	-4.47	-4.04	-2.44	6.81
	(0.25)	(0.28)	(0.29)	(0.35)	(0.19)	(0.75)
Wii	-0.59	1.68	-0.79	-1.97	-1.02	6.97
	(0.44)	(0.48)	(0.37)	(0.47)	(0.22)	(0.62)
Wii U	2.57	4.92	1.29	0.52	0.05	9.60
	(0.42)	(0.40)	(0.43)	(0.45)	(0.26)	(0.97)
Wii Wheel	-4.33	-3.36	-3.57	-4.07	-2.24	5.40
	(0.30)	(0.27)	(0.33)	(0.37)	(0.17)	(0.64)
Wii Motion	-3.23	-2.62	-2.99	-3.12	-1.86	6.03
	(0.25)	(0.24)	(0.25)	(0.29)	(0.15)	(0.59)

First, the Indep model, i.e., the version of the ALCM that constrains all utility interactions to be zero, infers all game consoles to be less attractive than when utility interactions are accounted for (compare Indep to ALCMhet, ALCMhetInv, and ALCMhom

Table 5: Heterogeneity in Price Sensitivity and Alternative Specific Constants - Hierarchical Prior Standard Deviations

	ALCMhet	ALCMhet-Inv	ALCMhom	Indep	MvP	IndepCPE
Price	0.33	0.35	0.30	0.31	0.10	0.66
	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.03)
Xbox 360	4.38	4.26	3.79	4.42	2.23	4.82
	(0.27)	(0.28)	(0.26)	(0.31)	(0.16)	(0.79)
Xbox One	6.42	6.50	5.20	5.77	2.76	7.49
	(0.50)	(0.41)	(0.42)	(0.51)	(0.21)	(0.79)
Xbox Kinect	2.68	2.32	2.65	2.99	1.72	5.46
	(0.25)	(0.24)	(0.23)	(0.30)	(0.15)	(0.68)
Xbox Wheel	2.72	2.34	2.77	3.27	1.81	4.38
	(0.29)	(0.21)	(0.25)	(0.36)	(0.15)	(0.91)
PS3	4.81	4.68	3.90	4.33	2.16	7.78
	(0.28)	(0.33)	(0.23)	(0.29)	(0.12)	(0.96)
PS4	8.71	9.25	7.43	8.29	3.81	11.86
	(0.48)	(0.52)	(0.55)	(0.56)	(0.21)	(0.99)
PS Eye	2.39	2.24	2.27	2.54	1.56	5.36
	(0.24)	(0.23)	(0.17)	(0.20)	(0.11)	(0.62)
PS Move	2.95	3.15	3.07	3.39	1.81	8.37
	(0.23)	(0.26)	(0.22)	(0.26)	(0.12)	(1.41)
PS Wheel	2.79	2.59	3.10	3.53	2.03	8.09
	(0.20)	(0.25)	(0.25)	(0.29)	(0.16)	(0.74)
Wii	5.44	5.44	4.94	6.00	2.74	7.58
	(0.43)	(0.31)	(0.38)	(0.51)	(0.23)	(0.73)
Wii U	5.62	6.25	5.22	6.03	2.83	7.90
	(0.37)	(0.40)	(0.41)	(0.47)	(0.21)	(1.01)
Wii Wheel	3.84	3.24	3.32	3.88	1.98	5.35
	(0.31)	(0.28)	(0.31)	(0.34)	(0.15)	(0.72)
Wii Motion	3.25	3.03	3.22	3.78	2.07	5.66
	(0.25)	(0.23)	(0.23)	(0.30)	(0.16)	(0.59)

in Table 4). Because Indep has no way to account for substitution between game consoles, it cannot but suppress the attractiveness of individual game consoles to rationalize the observed data. Second, after accounting for individual inventory (ALCMhetInv), we recover higher preferences for game consoles compared to versions of the ALCM that do not. The explanation is again related to substitution. To the extent that game consoles at home act as substitutes to game consoles offered in a menu, a model that does not account for this effect will infer lower average population preferences for game consoles in order to rationalize the observed data. Coefficients estimated by the MvP are absolutely smaller because of the difference in error normalization in the probit and the logit model. The coefficients estimated by IndepCPE cannot be directly compared because they refer to preferences at hypothetical (cross-)prices of zero. Table 5 summarizes heterogeneity, i.e., hierarchical prior standard deviations. The relatively larger amounts of preference-heterogeneity inferred for Xbox One and PS4, essentially across all models stand out as a substantively interesting result.

Table 6: Demand Interactions between Game Consoles

	ALCMhet	ALCMhet-Inv	ALCMhom	MvP	IndepCPE →	IndepCPE ←
Xbox 360-Xbox One	-4.91	-4.09	-2.74	-0.03	0	0
	(0.18)	(0.12)	(0.13)	(0.05)	(-)	(-)
Xbox 360-PS3	-4.82	-4.04	-1.1	-0.17	-0.41	-0.57
	(0.14)	(0.12)	(0.11)	(0.07)	(0.15)	(0.11)
Xbox 360-PS4	-4.86	-4.14	-1.08	-0.27	0	-0.66
	(0.15)	(0.13)	(0.1)	(0.06)	(-)	(0.05)
Xbox 360-Wii	-4.79	-4	-0.94	-0.02	-0.45	0
	(0.16)	(0.12)	(0.04)	(0.05)	(0.07)	(-)
Xbox 360-Wii U	-4.89	-4.07	-1.23	-0.09	0	0
	(0.17)	(0.12)	(0.13)	(0.06)	(-)	(-)
Xbox One-PS3	-4.93	-4.1	-0.97	-0.15	0	-0.17
	(0.17)	(0.12)	(0.05)	(0.08)	(-)	(0.03)
Xbox One-PS4	-4.96	-4.19	-1.72	-0.34	-0.45	0
	(0.16)	(0.12)	(0.04)	(0.08)	(0.04)	(-)
Xbox One-Wii	-4.91	-4.09	-0.95	-0.05	0	-0.41
	(0.17)	(0.12)	(0.08)	(0.04)	(-)	(0.08)
Xbox One-Wii U	-4.87	-4.07	-0.85	-0.06	0	-0.62
	(0.15)	(0.12)	(0.08)	(0.08)	(-)	(0.05)
PS3-PS4	-5.08	-4.32	-3.03	-0.36	-0.34	0
	(0.13)	(0.11)	(0.05)	(0.05)	(0.04)	(-)
PS3-Wii	-4.83	-4.09	-1.37	-0.24	-0.26	0
	(0.15)	(0.12)	(0.05)	(0.06)	(0.08)	(-)
PS3-Wii U	-4.97	-4.15	-1.66	-0.36	0	0
	(0.13)	(0.12)	(0.13)	(0.08)	(-)	(-)
PS4-Wii	-4.79	-4.04	-0.54	-0.12	0	0
	(0.16)	(0.1)	(0.05)	(0.08)	(-)	(-)
PS4-Wii U	-4.81	-3.96	-0.91	0	0	-0.5
	(0.16)	(0.11)	(0.06)	(0.07)	(-)	(0.1)
Wii-Wii U	-4.99	-4.13	-2.18	-0.15	-0.57	0
	(0.15)	(0.12)	(0.11)	(0.05)	(0.03)	(-)

Selected demand interactions and their different measures across different models are summarized in Tables 6 and 7. Table 6 reports demand interactions between game consoles. Table 7 reports demand interactions between game consoles and accessories of the same brand. For the heterogeneous versions of the ALCM we report posterior means of selected elements of $E_i(\Theta_i)$, where E_i denotes the expectation across respondents. In parentheses we report posterior standard deviations. As alternative measures for comparison, we report error correlations from the MvP as well as posterior means of average cross-price effects from IndepCPE. Because cross-price effects need not be symmetric, we report the effect from the offer on the left to the offer on the right in the first column of Tables 6 and 7 in one column, and the effect in the other direction in next column.[2]

All versions of the ALCM infer strongly negative demand interactions among different consoles on average (see columns 2 - 4 in Table 6). Interestingly, assuming homogeneity

[2]For the complete set of demand interactions and corresponding measures of heterogeneity (where applicable) across different models, see Tables 18 to 25 in the online Appendix.

Table 7: Demand Interactions between Game Consoles and Accessories of the Same Brand

	ALCMhet	ALCMhet-Inv	ALCMhom	MvP	IndepCPE →	IndepCPE ←
Xbox 360-Xbox Kinect	1.15	1.26	1.57	0.51	-0.57	-1.03
	(0.12)	(0.11)	(0.07)	(0.05)	(0.09)	(0.07)
Xbox 360-Xbox Wheel	1.23	1.35	2.32	0.45	-0.46	0
	(0.1)	(0.12)	(0.05)	(0.06)	(0.24)	(-)
Xbox One-Xbox Kinect	1.04	1.23	0.87	0.38	0	0
	(0.1)	(0.12)	(0.1)	(0.05)	(-)	(-)
Xbox One-Xbox Wheel	1.14	1.31	1.84	0.47	0	0
	(0.09)	(0.11)	(0.05)	(0.06)	(-)	(-)
PS3-PS Eye	0.95	1.16	0.43	0.09	0	0
	(0.12)	(0.11)	(0.08)	(0.06)	(-)	(-)
PS3-PS Move	1.02	1.16	0.89	0.21	0	0
	(0.09)	(0.1)	(0.06)	(0.07)	(-)	(-)
PS3-PS Wheel	1.06	1.18	1.19	0.2	-0.55	0
	(0.09)	(0.11)	(0.05)	(0.06)	(0.19)	(-)
PS4-PS Eye	1.15	1.28	1.41	0.4	0	-0.37
	(0.11)	(0.09)	(0.08)	(0.05)	(-)	(0.06)
PS4-PS Move	1.26	1.37	1.73	0.53	0	0
	(0.09)	(0.11)	(0.09)	(0.04)	(-)	(-)
PS4-PS Wheel	1.2	1.36	1.69	0.5	0	0
	(0.11)	(0.09)	(0.05)	(0.05)	(-)	(-)
Wii-Wii Wheel	1.02	1.17	0.57	0.39	0	0
	(0.1)	(0.1)	(0.09)	(0.05)	(-)	(-)
Wii-Wii Motion	1.18	1.33	1.22	0.48	0	0
	(0.1)	(0.1)	(0.05)	(0.04)	(-)	(-)
Wii U-Wii Wheel	1.11	1.26	0.94	0.39	0	-0.32
	(0.1)	(0.08)	(0.13)	(0.05)	(-)	(0.07)
Wii U-Wii Motion	1.25	1.35	2.01	0.51	0	-0.53
	(0.09)	(0.1)	(0.07)	(0.04)	(-)	(0.04)

in these interactions across respondents leads to relatively weaker interactions between consoles (see column "ALCMhom" versus columns 2 and 3 in Table 6). Although the error correlation in the MvP reflect the sign on of these negative interactions, only seven out of fifteen are credibly different from zero (see column five in Table 6). Finally, posterior means of selected average cross-price effects reported in columns 6 and 7 of Table 6 lack face validity.[3] All estimated cross-price effects between consoles are credibly negative on average, suggesting that this model is misspecified. We further characterize the nature of this misspecification in Appendix D.

In terms of interactions between consoles and accessories carrying the same brand, all models agree directionally (see Table 7). All versions of the ALCM infer credibly positive demand interactions on average. Similarly, all error correlations in the MvP but one (PS3-PS Eye), as well as all estimated cross-price effects in IndepCPE are credibly positive and negative, respectively.

[3]Cross-price effects to estimate in IndepCPE were chosen following the MBC industry guidelines (Orme, 2013). These guidelines suggest to retain cross-price effects based on the statistical significance of cross-price dependencies established in aggregate count analyses.

ALCMhet and ALCMhetInv both rely on a hierarchical prior distribution for individual utility interactions (see Subsection 3.2). Table 8 summarizes the posterior of the hierarchical prior parameters that characterize the distribution of θ-elements classified as connecting $i$ndependent (I), $s$ubstitutive (S), and $c$omplementary (C) offers. Table 9 reports posterior means of hierarchical prior classification probabilities for θ-elements into the three classes I, S, and C for both models. Together Tables 8 and 9 strongly suggest that utility interactions are relevant for choices from the menus in our experiment. For both models, the posterior means of $\bar{\theta}^{(S)}$ and $\bar{\theta}^{(C)}$ are sizable, relative to alternative specific constants in Table 4, and well separated from $\bar{\theta}^{(I)} \equiv 0$, even after taking heterogeneity (V_θ) into account.

The posterior means of hierarchical prior classification probabilities in Table 9 vary by (soft) rule, and in the expected directions. This supports the usefulness of these rules as a basis for probabilistically structuring the extremely high dimensional posterior of θ-elements. For example, an element $\theta_{i,k,k'}$ connecting two consoles in ALCMhet a priori belongs to the substitute class with a probability of about 76%, and an element connecting a console to an accessory of the same brand a priori belongs to the complement class with a probability of about 22%.

Table 8: Substitution and Complementarity - Posterior Means and Variance of Hierarchical Prior Parameters

Model	Means ($\bar{\theta}$)			Variance (V_θ)		
	I	S	C	I	S	C
ALCMhet	-0.01	-6.46	5.25	0.09	1.05	1.01
	(0.01)	(0.08)	(0.13)	(0.00)	(0.05)	(0.04)
ALCMhetInv	0.00	-5.40	4.67	0.08	0.98	1.02
	(0.01)	(0.04)	(0.09)	(0.00)	(0.04)	(0.04)

Table 9: Prior Classification Probabilities

	ALCMhet			ALCMhetInv		
	baseline	2 consoles	console own accessory	baseline	2 consoles	console own accessory
Prob of I in %	75.63	24.27	78.38	67.59	24.24	72.93
Prob of S in %	23.32	75.73	0	30.78	75.76	0
Prob of C in %	1.05	0	21.62	1.63	0	27.07

Finally, we report selected preference correlations in the population of respondents across different models. Table 10 summarizes preference correlations between game consoles, and Table 11 preference correlations between game consoles and accessories carrying

Table 10: Correlations in Individual Preferences for Game Consoles

	ALCMhet	ALCMhet-Inv	ALCMhom	Indep	MvP	IndepCPE
Xbox 360-Xbox One	0.45	0.57	0.51	0.38	0.36	0.92
	(0.07)	(0.08)	(0.08)	(0.1)	(0.08)	(0.03)
Xbox 360-PS3	0.87	0.92	0.81	0.74	0.67	0.93
	(0.02)	(0.02)	(0.03)	(0.04)	(0.05)	(0.03)
Xbox 360-PS4	0.4	0.56	0.3	0.05	0.1	0.92
	(0.07)	(0.09)	(0.08)	(0.09)	(0.08)	(0.05)
Xbox 360-Wii	0.63	0.72	0.54	0.49	0.44	0.92
	(0.06)	(0.05)	(0.07)	(0.07)	(0.07)	(0.04)
Xbox 360-Wii U	0.26	0.59	0.29	0.13	0.13	0.93
	(0.08)	(0.08)	(0.1)	(0.1)	(0.09)	(0.03)
Xbox One-PS3	0.32	0.57	0.35	0.12	0.19	0.96
	(0.1)	(0.07)	(0.09)	(0.1)	(0.09)	(0.02)
Xbox One-PS4	0.78	0.91	0.73	0.56	0.5	0.95
	(0.04)	(0.02)	(0.05)	(0.08)	(0.07)	(0.03)
Xbox One-Wii	0.1	0.23	-0.11	-0.22	0	0.96
	(0.12)	(0.1)	(0.1)	(0.13)	(0.09)	(0.02)
Xbox One-Wii U	0.63	0.76	0.49	0.4	0.39	0.96
	(0.06)	(0.04)	(0.08)	(0.1)	(0.09)	(0.02)
PS3-PS4	0.47	0.63	0.45	0.22	0.24	0.96
	(0.06)	(0.07)	(0.07)	(0.08)	(0.07)	(0.02)
PS3-Wii	0.59	0.65	0.52	0.45	0.4	0.96
	(0.06)	(0.06)	(0.07)	(0.07)	(0.06)	(0.02)
PS3-Wii U	0.24	0.51	0.21	0.02	0.06	0.96
	(0.09)	(0.07)	(0.1)	(0.1)	(0.09)	(0.01)
PS4-Wii	-0.02	0.15	-0.16	-0.27	-0.18	0.96
	(0.08)	(0.13)	(0.09)	(0.08)	(0.07)	(0.04)
PS4-Wii U	0.52	0.61	0.36	0.2	0.23	0.96
	(0.07)	(0.06)	(0.08)	(0.08)	(0.08)	(0.03)
Wii-Wii U	0.5	0.62	0.46	0.41	0.38	0.96
	(0.09)	(0.09)	(0.08)	(0.08)	(0.07)	(0.02)

the same brand. Posterior standard deviations are in parentheses.[4]

Table 10 shows that preference correlations between consoles are generally suppressed when substitution between consoles is not modeled (compare columns 2 - 4 [ALCMs] to columns 5 [Indep] and 6 [MvP]). A higher preference correlation between two consoles in the ALCMs can be explained by the fact that some respondents actually like both consoles more than other respondents (high β_i), but are unlikely to choose them together from a menu because of substitution. Because Indep and MvP do not (sufficiently) capture this substitution effect, these models compensate by biasing preference correlations downwards. For the same structural reason Indep and MvP in general overstate the preference correlation between consoles and console accessories carrying the same brand (see Table 11). Finally, the extreme preference correlation inferred by IndepCPE in column 7 likely are spurious results from extrapolating into a choice set where *all* prices

[4]See Tables 26 to 31 in the online Appendix for the complete set of preference correlations in all models.

Table 11: Correlations in Individual Preferences for Game Consoles and Accessories of the Same Brand

	ALCMhet	ALCMhet-Inv	ALCMhom	Indep	MvP	IndepCPE
Xbox 360-Xbox Kinect	0.19	0.28	0.29	0.53	0.42	0.9
	(0.11)	(0.12)	(0.08)	(0.07)	(0.08)	(0.04)
Xbox 360-Xbox Wheel	0.17	0.32	0.26	0.48	0.41	0.88
	(0.09)	(0.1)	(0.08)	(0.07)	(0.07)	(0.06)
Xbox One-Xbox Kinect	0.24	0.25	0.34	0.41	0.31	0.93
	(0.11)	(0.12)	(0.09)	(0.1)	(0.09)	(0.03)
Xbox One-Xbox Wheel	0.04	0.03	0.08	0.2	0.14	0.9
	(0.1)	(0.1)	(0.1)	(0.09)	(0.09)	(0.05)
PS3-PS Eye	0.19	0.27	0.28	0.31	0.23	0.94
	(0.11)	(0.08)	(0.08)	(0.08)	(0.07)	(0.02)
PS3-PS Move	0.2	0.27	0.23	0.28	0.23	0.96
	(0.08)	(0.1)	(0.08)	(0.08)	(0.07)	(0.02)
PS3-PS Wheel	0.06	0.21	0.17	0.24	0.16	0.96
	(0.09)	(0.09)	(0.08)	(0.08)	(0.07)	(0.02)
PS4-PS Eye	0.11	0.15	0.08	0.36	0.28	0.94
	(0.09)	(0.09)	(0.08)	(0.06)	(0.07)	(0.02)
PS4-PS Move	0.48	0.46	0.34	0.6	0.51	0.97
	(0.06)	(0.08)	(0.07)	(0.05)	(0.05)	(0.01)
PS4-PS Wheel	0.22	0.21	0.22	0.46	0.37	0.97
	(0.09)	(0.08)	(0.07)	(0.06)	(0.07)	(0.01)
Wii-Wii Wheel	0.48	0.51	0.44	0.53	0.46	0.92
	(0.06)	(0.07)	(0.07)	(0.06)	(0.07)	(0.05)
Wii-Wii Motion	0.59	0.65	0.6	0.69	0.61	0.94
	(0.06)	(0.06)	(0.05)	(0.04)	(0.05)	(0.02)
Wii U-Wii Wheel	0.31	0.41	0.37	0.53	0.43	0.93
	(0.09)	(0.08)	(0.08)	(0.07)	(0.07)	(0.04)
Wii U-Wii Motion	0.46	0.52	0.42	0.61	0.51	0.95
	(0.07)	(0.07)	(0.08)	(0.05)	(0.06)	(0.02)

are zero. Overall, it appears that if a model measures demand interdependencies incorrectly, or does not account for such interdependencies at all, correlations in individual preferences for substitutes will be underestimated, whereas correlations in preferences for complements will be overestimated.

4.4 Predictive Performance

Predictive performance is measured by hit rate (HR) and log predictive likelihood (LPL). We differentiate between marginal and menu HR. The former is defined as the proportion of correctly predicted choices of items or individual offers in a menu (marginal HR), whereas the later is defined as a proportion of correctly predicted choice combinations (menu HR). For example, if an individual chooses {A,B} out of a menu {A,B,C} and we predict {A-chosen, B-chosen, C-not chosen}, we declare it as three hits for marginal HR and as one hit for menu HR. However, if we predict e.g., {A-chosen, B-chosen, C-chosen}, we count this as two hits and one miss for marginal HR, and as a miss for menu HR, although the choice of {A,B} was predicted correctly. Since HR treats the case with predicted probability of 0.51 to be the same as 0.99, it is difficult to say which model is more precise in its prediction. LPL overcomes this problem. LPL of observing the data can be expressed as

$$LPL = \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} y_{i,t,k} log(\bar{\hat{p}}_{i,t,k}) + (1 - y_{i,t,k}) log(1 - \bar{\hat{p}}_{i,t,k}) \tag{23}$$

where $\bar{\hat{p}}_{i,t,k}$ is the estimated *marginal* probability of choosing item k in menu t by individual i, and $y_{i,t,k}$ represents the realized choice of item k in menu t by individual i. To obtain $\bar{\hat{p}}_{i,t,k}$, we compute the *marginal* probability $\hat{p}_{i,t,k}$ at each draw of the posterior distribution of parameters by Gibbs-sampling from the set of conditional probabilities that define the pseudo-likelihood (see Appendix A), and then take the average over posterior draws of parameters. Thus, we do not condition on holdout choices in any way making our predictions. The closer LPL to zero, the better the predictive performance of the model.

Table 12 summarizes predictive performance of the models we compare in our two holdout menus. We do not find substantial differences between models in marginal HR (1&0), which is in the range between 93 and 94%. This can be explained by the prevalence of zeros in each choice, i.e., only a relatively small number of items are chosen from each menu. Decomposing marginal HR into predictions of choices (marginal HR 1) and predictions of prevalent non-choices (marginal HR 0), we find that the heterogenous ALCMs outperform MvP in predicting marginal choices by about 4 percentage points. IndepCPE also performs well in terms of marginal HR 1, but is not on par with ALCMhet in terms of LPL. Overall, ALCMhet yields the most precise marginal predictions. As such, the LPL of ALCMhet is -1919, compared to, e.g., the LPL of IndepCPE at -2376, being the least precise model.

However, the crucial question in the analysis of data from MBCEs is how well a model predicts joint outcomes, i.e., actually chosen offer combinations. Column 5 in Table 12 (menu HR overall) shows that MvP, Indep and IndepCPE are not competitive in terms of menu HR, trailing the performance of ALCMs that account for heterogeneity in utility interactions by about 4 to 5%. An approximate standard error for the hit rates in this column is 0.017.

Interestingly, ALCMhom, which estimates homogeneous $\boldsymbol{\Theta}$ across individuals, performs relatively better too, suggesting that utility interactions in the ALCM capture demand interdependencies present in our data better than the error correlations in the multivariate probit model. Unfortunately, we cannot compute the LPL for our menu predictions because of the difficulty of evaluating the likelihood that motivated the use of the exchange algorithm for inference earlier.[5]

Table 12: Predictive Performance - Holdout Choices

Model	Marginal HR			Menu HR			
	1 & 0	1	0	overall	none	1 item	> 1 item
ALCMhet	0.936	0.585	0.980	0.575	0.876	0.512	0.468
	(-1919)	(-1182)	(-737)				
ALCMhetInv	0.937	0.586	0.981	0.578	0.876	0.544	0.446
	(-2138)	(-1451)	(-687)				
ALCMhom	0.938	0.555	0.985	0.555	0.891	0.532	0.391
	(-1921)	(-1193)	(-728)				
Indep	0.937	0.555	0.985	0.531	0.905	0.520	0.332
	(-1919)	(-1204)	(-714)				
MvP	0.937	0.539	0.986	0.538	0.896	0.517	0.357
	(-1928)	(-1220)	(-709)				
IndepCPE	0.935	0.590	0.977	0.528	0.836	0.538	0.346
	(-2376)	(-1727)	(-650)				

Finally, we assess how well different models predict menu outcomes, depending on how many items were chosen in a menu. Columns 6-8 of the Table 12 summarize the results and compare menu HRs for none-choices, one-item, and multiple-item choices. We find that whereas the baseline models are competitive in predicting none and one-item choices, they are clearly inferior in terms of predicting multiple-item purchases. For instance, ALCMhet predicts 46.8% of the multiple-item choices correctly, whereas MvP only 35.7%, resulting in a relative improvement of 30%. An approximate standard error for the multiple item choice hit rates is 0.027.

5 Pricing Implications

A simple model that ignores demand interdependencies might sometimes be sufficient to make good local predictions. The choice of the model becomes more important when a manager is to draw inferences about optimal actions. Next, we illustrate how optimal pricing patterns for complements and substitutes differ from those for independently valued products under different model assumptions. We use the posterior obtained under different ALCM versions, as well as that from baseline models, solve a given optimization

[5]When simulating menu-choices to estimate probabilities for the $2^{13} = 8192$ pontential choices from each menu as relative frequencies, there is always a chance that we obtain a probability of zero for a particular observed response because of the numerical constraints of simulation.

problem and compare the results. We find that accounting for substitution and complementarity among offers in a menu results in different optimal prices for products, in line with economic intuition.

For illustration, consider a monopolistic situation in which Sony optimizes prices for its products (PS3, PS4, PS Eye Cam, PS Move, and PS Wheel), given the competitor's products (Nintendo's Wii, Wii U, Wii Wheel, Wii Motion and Microsoft's Xbox 360, Xbox One, Xbox Kinect, Xbox Wheel) and prices in the market. Assuming marginal costs—set to zero without loss of generality—we define a five-dimensional grid of possible prices for Sony products ($[69, 95, 99, 105, 169]$ for PS3, $[199, 245, 249, 269, 299]$ for PS4, $[19, 39, 99, 119, 139]$ for PS Eye Cam, $[39, 59, 99, 109, 119]$ for PS Move, and $[29, 39, 79, 99, 119]$ for PS Wheel, all prices in Euros) and compute revenues for each price combination. We compute revenues for each individual in the sample based on posterior draws of individual parameters, $\boldsymbol{\beta}_i$ and $\boldsymbol{\Theta}_i$. We average over the draws of individual choice shares, compute individual revenues, then sum over all individuals, and, finally, find the price combination that maximizes the overall revenue.

Table 13 reports revenue maximizing price vectors from different models. Indep and MvP, which do not account for utility interactions, price accessories too high, relative to the optimal prices for accessories from the different ALCMs. IndepCPE, which includes cross-price effects, yields even higher optimal prices for PS Eye Cam and PS Move. We also see that ALCMs, and especially the two ALCMs with a structured account of heterogeneity in utility interactions yield relatively higher optimal prices for the two consoles PS3 and PS4.

Table 13: Optimal Pricing

Model	PS3	PS4	Eye Cam	Move	Wheel
ALCMhet	105	245	39	59	39
ALCMhetInv	105	245	39	59	39
ALCMhom	99	245	39	59	39
Indep	95	245	99	99	79
MvP	95	245	119	99	119
IndepCPE	69	199	139	119	29

The pattern in the optimal price vectors across different models showcases an important managerial implication of accounting for substitution and complementarity. The complementarity between accessories and game consoles captured by the ALCMs leads to lower prices for accessories. The relatively higher prices for consoles from these models—despite the complementarity to accessories—comes through strong substitution between

PS3 and PS4. Whereas Indep and MvP predict that lower prices for PS3 and PS4 may likely result in a joint purchase, the ALCMs correctly recognize that this is an outlying event.

Table 14 reports market shares at revenue maximizing prices. The first 6 lines in Table 14 show market shares computed using the ALCMhet, in turn conditioned on revenue maximizing prices from the different models fitted. The following lines report shares computed using the models indicated in the first column at the optimal prices implied by that same model.

High optimal accessory prices recommended by the baseline models Indep and MvP (see Table 13) result in much lower market shares for accessories compared to those obtained from optimization based on the ALCMs in Table 14. However, Indep and MvP tend to underestimate these shares (see lines "Indep" and "MvP"), presumably because these models do not account for the complementarity between consoles and accessories. IndepCPE suggests extremely high prices for Eye Cam and Move and relatively low prices for the two consoles (see Table 13). It vastly overestimates the shares of PS4 and Move at these prices (compare lines "ALCMhet at IndepCPE" and "IndepCPE" in Table 14). Finally, if a manager sets optimal prices suggested by IndepCPE, MvP, and Indep in this scenario, the loss in revenues relative to ALCMhet is 5.65%, 3.53%, and 2.10%, respectively.

Table 14: Market Shares

Model	PS3	PS4	Eye Cam	Move	Wheel
ALCMhet	17.4	35.0	10.2	19.6	11.3
ALCMhet at ALCMhetInv	17.4	35.0	10.2	19.6	11.3
ALCMhet at ALCMhom	19.7	34.6	10.5	19.8	11.4
ALCMhet at Indep	17.8	33.1	5.0	13.0	7.0
ALCMhet at MvP	17.4	32.8	4.1	13.0	4.4
ALCMhet at IndepCPE	22.3	39.8	3.7	11.4	13.7
ALCMhetInv	17.0	34.8	9.8	19.0	11.1
ALCMhom	17.8	34.2	9.5	18.6	10.7
Indep	17.6	33.7	3.8	9.8	5.3
MvP	16.6	33.2	3.7	9.9	4.1
IndepCPE	21.6	43.6	4.1	21.0	16.5

Next, we demonstrate how different models react to price changes in the market. We cut the prices of the competitive Microsoft and Nintendo offers in half and reoptimize the prices for Sony products following the procedure described earlier. Table 15 summarizes the resulting price changes. For example, before the competitive price cut, ALCMhet suggested a price of 245 for PS4 (see Table 13). After the competitive price cut, ALCMhet suggests to lower this price by 46 to 199.

ALCMhet, ALCMhetInv and ALCMhom imply to lower prices for PS game consoles, and especially those of PS4. Whith the exception of PS Move under ALCMhom, accessory prices are not adjusted. IndepCPE reacts to the price changes too, but only by a small adjustment to the price of PS3. By definition, optimal prices are independent of competitors' prices in Indep that sets all demand interdependencies to zero and thus optimal prices stay the same. However, it may come as a surprise that the same holds for the MvP, as explained analytically in Section 2.2 (see Equation 10). Finally, if a manager sets optimal prices suggested by IndepCPE, MvP, and Indep under this competitive scenario, the loss in revenues relative to ALMChet amounts to 5.61 %, 6.15%, and 5.13%, respectively.

Table 15: Optimal Pricing - Reaction to Price Changes in the Market (Δp)

Model	PS3	PS4	Eye Cam	Move	Wheel
ALCMhet	-10	-46	0	0	0
ALCMhetInv	-36	-46	0	0	0
ALCMhom	-4	-46	0	-10	0
Indep	0	0	0	0	0
MvP	0	0	0	0	0
IndepCPE	-10	0	0	0	0

Thus, if and in what way demand interdependencies are modeled matters for pricing decisions. Given that the ALCM, especially when coupled with a structured account of heterogeneity, is better supported by the data than Indep, MvP or IndepCPE, and the intuitive differences between optimal price vectors from different models, we conclude that an ALCM coupled with a structured account of heterogeneity will result in better pricing decisions in this and similar applications.

6 Discussion

Studies that involve choices from menus have been gaining increased popularity. Given that consumers often face menu-like purchase situations and are commonly allowed to customize products and services, this is not surprising. Consumers' decisions in this case extend beyond the well studied situation of choice among perfect substitutes.

In this paper, we propose the ALCM as a utility-based framework to handle the combination of substitutes and complements likely to be present in many menus.

The parsimonious but flexible prior structure we develop captures extreme heterogeneity in two-way utility interactions. Such extreme heterogeneity in the form of 'infinite differences' between consumers occur in populations where some consumers perceive two offers to be perfect substitutes and others view the same two offers as horizontally differentiated.

We illustrate our methodology through an application to data from an industry grade MBCE study eliciting demand for game consoles and accessories, designed by GfK for academic purposes. We find strong empirical evidence of substitution and complementarity between offers in our experiment. In this context we show how to use information about respondents' inventory as conditioning argument that exerts its influence on observed choices via substitution or complementarity with inside offers. We believe that this is a more theoretically appealing way of bringing knowledge about inventory into the model than using inventory as a covariate to inherent preferences.

We show that approximations to utility interactions based on correlated errors as in the multivariate probit model, or through cross-price effects result in inferior predictive fit, especially when predicting choices that comprise more than one menu item. Importantly, we document that the inferences from extant models result in counterintuitive pricing decisions and lower revenues.

We trace the worse performance of extant models, i.e., the multivariate probit and the binomial logit model that includes cross-prices effects to the failure of *generating* important implications from the economics of substitution and complementarity. Specifically, we show analytically that the error correlations in the multivariate probit model can *reflect* substitution and complementarity between offers in a menu. However, when the indirect utility of substitutes and complements changes, error correlations fail at *generating* the implied change in marginal shares of a target offering.

The binomial logit model that includes cross-price effects attempts to capture substitution through positive, and complementarity through negative cross-price effects. However, substitution and complementarity exert their influence on individual choices even in the absence of price variation. And in those instances where price variation is sufficient for cross-price effects to pick up and *reflect* substitutive and complementary relationships, cross-price effects fail at *generating* substitution between offers whenever prices of substitutes equate their indirect utilities.

On the technical side we develop Bayesian inference for the ALCM leveraging the recently proposed exchange algorithm in combination with perfect sampling of data from the ALCM to overcome the problem of a computationally intractable normalizing constant in the likelihood. We also develop an auxiliary data augmentation scheme to enable effective joint proposals of highly correlated parameters while maintaining the detailed balance conditions.

While we believe that the advantages of the ALCM over extant models for choices from menus are apparent from our paper, there is certainly room for additional development. First, like all extant models in this context, the suggested modeling framework ignores income effects. Second, the ALCM in its current implementation is limited to two-way utility interactions. Depending on the application, selected higher order interactions may also be important. However, the development of a prior that adaptively selects relevant, potentially heterogenous higher order interactions involves substantial theoretical and algorithmic development, given the enormous search space implied that we leave to future research.

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A Sampling $\mathbf{Y}_i^c$

The exchange algorithm requires that we sample from Equation 1. Direct sampling requires evaluating the normalizing constant $nc_t\left(\boldsymbol{\vartheta}_i^c\right)$ which the exchange algorithm is designed to avoid.

One way to obtain draws of $\mathbf{y}_{i,t}$ without evaluating $nc_t\left(\boldsymbol{\vartheta}_i^c\right)$ is to 'Gibbs-through' the conditional distributions defined in Equation 3. Because these conditional distributions are in binomial logit form, probabilities can be computed fast. The Gibbs-sampler updates elements in $\mathbf{y}_{i,t} = (y_{i,t,1}, \ldots, y_{i,t,k}, \ldots, y_{i,t,K})$ one at a time conditional on $\mathbf{y}_{i,t,-k}$.

1. Initialize $\mathbf{y}_{i,t}^{(m-1)}$ as the observed vector of choices.
2. Sample from $y_{i,t,k} \sim Bernoulli\left(P(y_{i,t,k}|\vartheta_i^c, \mathbf{y}_{i,t,-k}^{(m-1)})\right)$
3. Replace the k-th element in $\mathbf{y}_{i,t}^{(m-1)}$ by the draw just obtained and proceed to generating $y_{i,t,k+1}$ as in step number 2. Repeat steps 2 and 3 until all K elements in $\mathbf{y}_{i,t}^{(m-1)}$ have been updated. Then set $\mathbf{y}_{i,t}^{(m)} = \mathbf{y}_{i,t}^{(m-1)}$. This completes one Gibbs cycle.
4. Return to step 2 until convergence from the initial condition.

A drawback of this approach is that it remains unclear how often the Gibbs-cylce needs to be repeated before a draw equivalent to direct sampling from Equation 1 is obtained. An elegant way around this problem is the perfect sampler proposed by Propp and Wilson (1996).

In our case study we rely on the exchange algorithm in combination with perfect sampling from Equation 1. In simulation studies not reported here, we found both Gibbs-sampling and perfect sampling of $\mathbf{Y}_i^c$ to work in practice. However, we prefer not to worry about the number of Gibbs-cycles required for convergence to $\mathbf{y}_{i,t}^c \sim P_t(\vartheta_i^c)$. Nevertheless, the Gibbs-sampler may be the better choice when the goal is to estimate probabilities, which requires larger samples of $\mathbf{y}_{i,t}$, because each individual draw from the perfect sampler is more computationally expensive. We use the Gibbs-sampler when 'simulating forward' from the model for predictive validation and to compute counterfactual results in our case study.

B Detailed Balance in the Exchange Algorithm

For later reference, note that the normalized posterior is defined as

$$p\left(\boldsymbol{\vartheta}_i|\mathbf{Y}_i\right)=\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)}p\left(\boldsymbol{\vartheta}_i\right)\left(p\left(\mathbf{Y}_i\right)\right)^{-1} \tag{24}$$

where

$$p\left(\mathbf{Y}_i\right)=\int\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)}p\left(\boldsymbol{\vartheta}_i\right)d\boldsymbol{\vartheta}_i \tag{25}$$

The detailed balance condition requires that

$$\begin{aligned} &p\left(\boldsymbol{\vartheta}_i|\mathbf{Y}_i\right)q\left(\boldsymbol{\vartheta}_i^c\right)P\left(\mathbf{Y}_i^c|\boldsymbol{\vartheta}_i^c\right)\alpha\left(\boldsymbol{\vartheta}_i,\mathbf{Y}_i\rightarrow\boldsymbol{\vartheta}_i^c,\mathbf{Y}_i^c\right)\\ &\quad=p\left(\boldsymbol{\vartheta}_i^c|\mathbf{Y}_i\right)q\left(\boldsymbol{\vartheta}_i\right)P\left(\mathbf{Y}_i^c|\boldsymbol{\vartheta}_i\right)\alpha\left(\boldsymbol{\vartheta}_i^c,\mathbf{Y}_i\rightarrow\boldsymbol{\vartheta}_i,\mathbf{Y}_i^c\right)\end{aligned} \tag{26}$$

Substituting the right hand side of Equation 13 into the left hand side of Equation 26 we obtain

$$\begin{aligned} &p\left(\boldsymbol{\vartheta}_i|\mathbf{Y}_i\right)q\left(\boldsymbol{\vartheta}_i^c\right)P\left(\mathbf{Y}_i^c|\boldsymbol{\vartheta}_i^c\right)\alpha\left(\boldsymbol{\vartheta}_i,\mathbf{Y}_i\rightarrow\boldsymbol{\vartheta}_i^c,\mathbf{Y}_i^c\right)\\ &\quad=p\left(\boldsymbol{\vartheta}_i|\mathbf{Y}_i\right)q\left(\boldsymbol{\vartheta}_i^c\right)P\left(\mathbf{Y}_i^c|\boldsymbol{\vartheta}_i^c\right)min\left(1,\frac{p\left(\boldsymbol{\vartheta}_i^c\right)q\left(\boldsymbol{\vartheta}_i\right)}{p\left(\boldsymbol{\vartheta}_i\right)q\left(\boldsymbol{\vartheta}_i^c\right)}\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}\right)\\ &\quad=\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)}p\left(\boldsymbol{\vartheta}_i\right)\left(p\left(\mathbf{Y}_i\right)\right)^{-1}q\left(\boldsymbol{\vartheta}_i^c\right)\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)}\\ &\qquad\times min\left(1,\frac{p\left(\boldsymbol{\vartheta}_i^c\right)q\left(\boldsymbol{\vartheta}_i\right)}{p\left(\boldsymbol{\vartheta}_i\right)q\left(\boldsymbol{\vartheta}_i^c\right)}\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}\right)\\ &\quad=min\left(\begin{array}{c} p\left(\boldsymbol{\vartheta}_i\right)\left(p\left(\mathbf{Y}_i\right)\right)^{-1}q\left(\boldsymbol{\vartheta}_i^c\right)\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)}\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)},\\ p\left(\boldsymbol{\vartheta}_i^c\right)\left(p\left(\mathbf{Y}_i\right)\right)^{-1}q\left(\boldsymbol{\vartheta}_i\right)\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)}\prod_{t=1}^{T}\frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)}\end{array}\right)\end{aligned} \tag{27}$$

Going into the 'opposite direction' we obtain

$$
\begin{aligned}
&p\left(\boldsymbol{\vartheta}_i^c|\mathbf{Y}_i\right) q\left(\boldsymbol{\vartheta}_i\right) P\left(\mathbf{Y}_i^c|\boldsymbol{\vartheta}_i\right) \alpha\left(\boldsymbol{\vartheta}_i^c, \mathbf{Y}_i \rightarrow \boldsymbol{\vartheta}_i, \mathbf{Y}_i^c\right) \\
&\quad = p\left(\boldsymbol{\vartheta}_i^c|\mathbf{Y}_i\right) q\left(\boldsymbol{\vartheta}_i\right) P\left(\mathbf{Y}_i^c|\boldsymbol{\vartheta}_i\right) min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i\right) q\left(\boldsymbol{\vartheta}_i^c\right)}{p\left(\boldsymbol{\vartheta}_i^c\right) q\left(\boldsymbol{\vartheta}_i\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}\right) \\
&\quad = \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)} p\left(\boldsymbol{\vartheta}_i^c\right) \left(p\left(\mathbf{Y}_i\right)\right)^{-1} q\left(\boldsymbol{\vartheta}_i\right) \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)} \\
&\qquad \times min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i\right) q\left(\boldsymbol{\vartheta}_i^c\right)}{p\left(\boldsymbol{\vartheta}_i^c\right) q\left(\boldsymbol{\vartheta}_i\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}\right) \\
&\quad = min\left(\begin{array}{c} p\left(\boldsymbol{\vartheta}_i^c\right) \left(p\left(\mathbf{Y}_i\right)\right)^{-1} q\left(\boldsymbol{\vartheta}_i\right) \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)}, \\ p\left(\boldsymbol{\vartheta}_i\right) \left(p\left(\mathbf{Y}_i\right)\right)^{-1} q\left(\boldsymbol{\vartheta}_i^c\right) \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)}{nc_t\left(\boldsymbol{\vartheta}_i^c\right)} \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}{nc_t\left(\boldsymbol{\vartheta}_i\right)} \end{array}\right)
\end{aligned}
\tag{28}
$$

Because the last lines of Equations 27 and 28 are equal by construction, the exchange algorithm delivers detailed balance with respect to the normalized posterior distribution.

C MCMC Sampling

0. Initialize $\boldsymbol{\beta}_i$, $\boldsymbol{\Theta}_i$, for all $i = 1, \ldots, N$, and $\boldsymbol{\delta}$ at zero, and σ_ϵ at 1.

1. Draw from $\mathbf{z}_i|\boldsymbol{\delta}, \sigma_\epsilon, \mathbf{W}, ind_i$ for all $i = 1, \ldots, N$ using the inverse CDF-transformation. Here ind_i refers to a vector of classifications of elements in $\boldsymbol{\Theta}_i$ into S, I, or C initialized randomly (see Equation 16).

2. Draw from $\boldsymbol{\delta}, \sigma_\epsilon|\{\mathbf{z}_i\}, \mathbf{W}, prior$ using standard conjugate results and weakly informative priors $\boldsymbol{\delta} \sim N(0, \boldsymbol{A}^{-1})$ and $\sigma_\epsilon \sim IG(\nu, s)$, where $\boldsymbol{A} = 0.01$, $\nu = 3$, $s = 1$.

3. Compute prior classification probabilities $\mathbf{P}(S, I, C)|\boldsymbol{\delta}, \sigma_\epsilon, \mathbf{W}$, see Equations 18 through 22.

4. Draw from $\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta|\{\boldsymbol{\beta}_i\}, prior$. We use standard conjugate results to update hyperparameters $\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta$ based on weakly informative priors:

$$
\begin{aligned}
\bar{\boldsymbol{\beta}} &\sim N(0, \boldsymbol{A}^{-1}) \\
\boldsymbol{V}_\beta &\sim IW(\nu, \boldsymbol{V})
\end{aligned}
\tag{29}
$$

where $\boldsymbol{A} = 0.01$, $\nu = m + 3$, $\boldsymbol{V} = \nu\boldsymbol{I}$, and m is the number of elements in $\bar{\boldsymbol{\beta}}$.

5. Draw from $\bar{\boldsymbol{\theta}}, \boldsymbol{V_\theta}|\{ind_i, \boldsymbol{\theta}_i\}, prior$. See Section 3.2 for the subjective prior parameter setting. We again rely on standard conjugate results.

6. Draw from $\boldsymbol{\vartheta}_i, ind_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V_\theta}, \mathbf{P}(S, I, C), \mathbf{Y}_i$. This step employs the MH algorithm for the joint update of individual parameters, $\boldsymbol{\vartheta}_i = (\boldsymbol{\beta}_i, \boldsymbol{\Theta}_i)$ and ind_i, where ind_i refers to the vector of classifications of (unique) individual elements $\theta_{i,k,k'}$ in $\boldsymbol{\Theta}_i$. A joint update is required because given the classification of an element $\theta_{i,k,k'}$ into S, I, or C, the hierarchical prior $\theta_{i,k,k'}$ is highly informative.

   (a) Propose a classification *candidate* from $ind_i^c|\mathbf{P}(S, I, C), p^*$, where p^* is the probability of attempting the reclassification of a (unique) individual elements $\theta_{i,k,k'}$ in $\boldsymbol{\Theta}_i$. This probability is independently generated as $p^* \sim Beta(1, 5)$ at each draw for each respondent, and corresponds to a stochastically determined step-size. If an element of ind_i becomes a candidate for a re-classification attempt, the proposal is generated from its hierarchical prior $\mathbf{P}(S, I, C)_{k,k'}$ as a function of $\boldsymbol{\delta}, \sigma_\epsilon$ and $\mathbf{w}_{k,k'}$ (see Equations 18 through 22). In turn, ind_i then determines the structure of the prior for $\boldsymbol{\Theta}_i$.

   (b) Propose a *candidate* value $\boldsymbol{\vartheta}_i^c$ from $\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V_\theta}, ind_i^c, \mathbf{Y}_i$ using auxiliary data augmentation. The resulting proposal ensures concordance between ind_i^c and $\boldsymbol{\vartheta}_i^c$ which facilitates the large jumps in the parameters space implied by θ-reclassifications.

   We generate auxiliary latent variables $\zeta_{i,t,k}$ for offer k in menu t for each individual respondent i from a density denoted by $h(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i, \mathbf{Y}_i)$ that we explain next. Depending on whether the offer was chosen $(y_{i,t,k} = 1)$ or rejected $(y_{i,t,k} = 0)$, we generate $\zeta_{i,t,k}$ by truncating below (above) zero a t-distributed variable with 10 degrees of freedom, mean $\beta_{i,k} + \beta_{price,i}p_{t,k} + \mathbf{y'}_{i,t}\boldsymbol{\Theta}_i^k$, and variance $\frac{\pi^2}{3}$, where $\boldsymbol{\Theta}_i^k$ corresponds to the k-th column of $\boldsymbol{\Theta}_i$ (see Equation 7), such that $\mathbf{y'}_{i,t}\boldsymbol{\Theta}_i^k$ measures the contribution of $\mathbf{y'}_{i,t,-k}$ to the conditional probability $P(y_{i,t,k}|\boldsymbol{\vartheta}_i, \mathbf{y}_{i,t,-k})$. The auxiliary latent variables can be thought of as approximate data augmentation for the PL which implies logistically distributed latent variables (see Equations 3 and 15).

   We then regress the auxiliary variables $\zeta_{i,k,t}$ on offer specific constants and prices, and the respective $\mathbf{y'}_{i,t,-k}$ as illustrated next using a three-offer menu as example. With three offers in total, $\boldsymbol{\Theta}_i$ contains three unique θ-elements:

$$\boldsymbol{\Theta}_i = \begin{pmatrix} 0 & \theta_{i,1,2} & \theta_{i,1,3} \\ \theta_{i,2,1} & 0 & \theta_{i,2,3} \\ \theta_{i,3,1} & \theta_{i,3,2} & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \theta_{i,2,1} \\ \theta_{i,3,1} \\ \theta_{i,3,2} \end{pmatrix}$$

The regression equation then is

$$\begin{pmatrix} \zeta_{i,1,1} \\ \zeta_{i,1,2} \\ \zeta_{i,1,3} \\ \vdots \\ \vdots \\ \zeta_{i,T,1} \\ \zeta_{i,T,2} \\ \zeta_{i,T,3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & p_{1,1} & y_{i,1,2} & y_{i,1,3} & 0 \\ 0 & 1 & 0 & p_{1,2} & y_{i,t,1} & 0 & y_{i,1,3} \\ 0 & 0 & 1 & p_{1,3} & 0 & y_{i,1,1} & y_{i,1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & p_{T,1} & y_{i,T,2} & y_{i,T,3} & 0 \\ 0 & 1 & 0 & p_{T,2} & y_{i,T,1} & 0 & y_{i,T,3} \\ 0 & 0 & 1 & p_{T,3} & 0 & y_{i,T,1} & y_{i,T,2} \end{pmatrix} \begin{pmatrix} \beta_{i,1} \\ \beta_{i,2} \\ \beta_{i,3} \\ \beta_{i,price} \\ \theta_{i,2,1} \\ \theta_{i,3,1} \\ \theta_{i,3,2} \end{pmatrix} + \begin{pmatrix} \epsilon_{i,1,1} \\ \epsilon_{i,1,2} \\ \epsilon_{i,1,3} \\ \vdots \\ \vdots \\ \epsilon_{i,T,1} \\ \epsilon_{i,T,2} \\ \epsilon_{i,T,3} \end{pmatrix}$$

The first and the central second 'OLS-moments' of $\boldsymbol{\vartheta}_i$ from this regression are combined with the multivariate normal hierarchical prior $p(\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i^c)$ implied by the proposed classifications in ind_i^c to derive the location and scale parameter of a multivariate T proposal distribution with 10 degrees of freedom denoted $g(\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i^c, \boldsymbol{\zeta}_i)$.

(c) Finally, we substitute into Equation 13 to obtain the probability of accepting the joint move from $(\boldsymbol{\vartheta}_i, ind_i)$ to $(\boldsymbol{\vartheta}_i^c, ind_i^c)$:

$$\begin{aligned} &\alpha\left(\boldsymbol{\vartheta}_i, ind_i, \mathbf{Y}_i \rightarrow \boldsymbol{\vartheta}_i^c, ind_i^c, \mathbf{Y}_i^c\right) = \\ &min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i^c|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i^c\right) P(ind_i^c|\mathbf{P}(S,I,C))}{p\left(\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i\right) P(ind_i|\mathbf{P}(S,I,C))} \times \right. \\ &\quad \frac{g\left(\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i, \boldsymbol{\zeta}_i\right) h\left(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i^c, \mathbf{Y}_i\right) P(ind_i(\boldsymbol{I}(p^*))|\mathbf{P}(S,I,C))}{g\left(\boldsymbol{\vartheta}_i^c|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i^c, \boldsymbol{\zeta}_i\right) h\left(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i, \mathbf{Y}_i\right) P(ind_i^c(\boldsymbol{I}(p^*))|\mathbf{P}(S,I,C))} \times \\ &\quad \left. \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)} \right) \end{aligned} \tag{30}$$

The vertically separated factors on the left hand side of Equation 30 in turn correspond to the (hierarchical) prior of $(\boldsymbol{\vartheta}_i, ind_i)$ (see Equations 17 and 29, and Equations 18 through 22), the proposal distributions just described, and the likelihood contribution. The factors $h(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i^c, \mathbf{Y}_i)$ and $(h(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i, \mathbf{Y}_i))^{-1}$ in the third line of Equation 30 ensure the detailed balance between the move from the current state $(\boldsymbol{\vartheta}_i, ind_i, \mathbf{Y}_i)$ — via $\boldsymbol{\zeta}_i$ — to the candidate state, $(\boldsymbol{\vartheta}_i^c, ind_i^c, \mathbf{Y}_i^c)$, and the move in the opposite direction.

Finally, the notation $(\boldsymbol{I}(p^*))$ indicates the subset of elements in ind_i eligible for a potential reclassification in a particular iteration of the MCMC. All other elements in ind_i deterministically retain their old value in this iteration. Because we use the hierarchical prior as proposal, and elements in ind_i are independently distributed a priori, Equation 30 simplifies to:

$$\alpha\left(\boldsymbol{\vartheta}_i, ind_i, \mathbf{Y}_i \rightarrow \boldsymbol{\vartheta}_i^c, ind_i^c, \mathbf{Y}_i^c\right) =$$
$$min\left(1, \frac{p\left(\boldsymbol{\vartheta}_i^c|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i^c\right)}{p\left(\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i\right)} \frac{g\left(\boldsymbol{\vartheta}_i|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i, \boldsymbol{\zeta}_i\right) h\left(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i^c, \mathbf{Y}_i\right)}{g\left(\boldsymbol{\vartheta}_i^c|\bar{\boldsymbol{\beta}}, \boldsymbol{V}_\beta, \bar{\boldsymbol{\theta}}, \boldsymbol{V}_{\boldsymbol{\theta}}, ind_i^c, \boldsymbol{\zeta}_i\right) h\left(\boldsymbol{\zeta}_i|\boldsymbol{\vartheta}_i, \mathbf{Y}_i\right)} \times \right.$$
$$\left. \prod_{t=1}^{T} \frac{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i^c\right)}{\ell^*\left(\mathbf{y}_{i,t}|\boldsymbol{\vartheta}_i\right)} \frac{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i\right)}{\ell^*\left(\mathbf{y}_{i,t}^c|\boldsymbol{\vartheta}_i^c\right)}\right) \tag{31}$$

D IndepCPE

In our empirical study we estimate credibly negative cross-price effects between products that are clearly identified as strong substitutes on average in the population by the ALCM (see Section 4).

To numerically illustrate the misspecification inherent to the IndepCPE that causes this result, we simulate 100 choices from two-item menus comprising alternatives A and B from the ALCM with price parameter equal to -1 and alternative specific constants equal to 3 and 5 such that B is the more preferred brand overall. The utility interaction parameter θ is set to -10 implying relatively strong substitution between A and B given the magnitude of alternative specific constants.

Across the 100 two-item menus, prices of alternative A vary uniformly on the grid $[0.2, 0.4, 0.6, \ldots, 4]$. Prices of alternative B are constructed according to $p_B = p_A + 2$, such that the expected indirect utilities of both alternatives are exactly equal in each of the 100 menus. Thus, our simulated respondent is, before realizations of the error draws, always indifferent between A and B.

We then create additional data sets using the same setting except for how we generate B's prices. In these data sets we generate B's prices as $p_B = p_A + 2 + \xi$, where ξ is distributed iid across menus according to $\xi \sim N(0, \sigma)$. The ξ-draws serve to break the perfect utility balance between A and B.

Table 16: IndepCPE - Simulated Data

Noise	Posterior Means (std)				
	β_p	β_A	β_B	β_{pB}	β_{pA}
0	0.253	-0.337	0.086	-0.178	-0.517
	(0.857)	(1.772)	(1.734)	(0.848)	(0.888)
0.1	-0.096	-0.081	0.118	0.008	-0.267
	(0.715)	(1.517)	(1.483)	(0.711)	(0.754)
0.2	-0.101	0.950	-0.208	-0.272	0.009
	(0.605)	(1.349)	(1.288)	(0.607)	(0.634)
0.3	-0.175	0.884	0.030	-0.219	0.043
	(0.471)	(1.116)	(1.016)	(0.472)	(0.515)
0.4	-0.308	0.222	0.390	0.009	0.122
	(0.377)	(0.917)	(0.904)	(0.373)	(0.407)
0.5	-0.554	-0.369	0.889	0.275	0.408
	(0.333)	(0.841)	(0.797)	(0.322)	(0.384)
0.9	-0.773	-0.388	1.182	0.398	0.607
	(0.219)	(0.662)	(0.657)	(0.208)	(0.277)
2	-1.010	-0.440	1.087	0.523	0.882
	(0.164)	(0.526)	(0.610)	(0.124)	(0.282)
random	-1.327	-1.722	2.054	1.127	1.153
	(0.231)	(0.805)	(0.693)	(0.297)	(0.293)

Table 16 summarizes posterior means (standard deviations) of IndepCPE parameters, i.e., the price coefficient, alternative specific constants, the cross effect from B's price on the indirect utility of A, and vice versa. The rows in Table 16 correspond to different data sets and the first column indicates the standard deviation of the ξ-displacement of B's price in the data set. Thus, the first row corresponds to the case where the respondent always is exactly indifferent between A and B. The last row represents the case where we draw p_A and p_B completely randomly from the predefined set of prices. We see that IndepCPE fails to recover positive cross-price effects when a respondent is indifferent or about indifferent between A and B. As the noise in p_B increases, estimated cross-price effects become more positive and the uncertainty in parameters decreases. However, even when the price variation in the data is sufficient for cross-price effects to *reflect* substitution between A and B, the model fails at *generating* substitutive effects in individual menus where prices make the respondent indifferent between the two brands.

Next we illustrate the performance of the IndepCPE as an approximating model in a standard hierarchical setting with manipulated price variation as described above. We generate 2000 respondents with individual alternative specific constants distributed iid $\beta_i \sim N(5, 4)$ and price-sensitivity $\beta_{ip} \sim N(-1.5, 0.4)$, each solving 2 choice tasks comprised of alternatives A and B. We mimic the distribution of demand interdependencies recovered with ALCMhet in our case study and generate individual choices from the ALCM, with A and B being weak complements, weak substitutes or substitutes, with θ_i from the set $\{1.2, -1.5, -5\}$ with probabilities $(0.18, 0.63, 0.19)$. We draw prices from the grid $[1.1, 1.2, 1.3, \ldots, 5]$ such that individuals are about indifferent between the two offers ($\xi \sim N(0, 0.1)$) and fit the IndepCPE with a multivariate normal hierachical prior distribution over parameters to the data.

Table 17: Hierarchical Prior Means and Standard Deviations

	Mean	Standard Deviation
Price Sensitivity	-0.723	0.480
	(0.042)	(0.037)
β_A	3.747	1.725
	(0.229)	(0.414)
β_B	3.991	1.585
	(0.252)	(0.501)
β_{p_B}	-0.732	0.478
	(0.052)	(0.037)
β_{p_A}	-0.785	0.519
	(0.058)	(0.049)

Table 17 reports posterior means of the hyper parameters in the IndepCPE. Posterior standard deviations are in parentheses. Although substitutes prevail in the data set, inferred cross-price effects are credibly negative. These results illustrate how IndepCPE may fail at correctly *reflecting* substition and complementarity in applications.

Online Appendix: Posterior Means and Covariances of Utility Interactions, Correlations, Cross-Price Effects and Preference Correlations from Different Models

Table 18: Posterior Means of Individual Θ (std) - ALCMhet

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox One	-4.91 (0.18)	0										
Xbox Kinect	1.15 (0.12)	1.04 (0.1)	0									
Xbox Wheel	1.23 (0.1)	1.14 (0.09)	-1.44 (0.13)	0								
PS3	-4.82 (0.14)	-4.93 (0.17)	-1.45 (0.12)	-1.52 (0.13)	0							
PS4	-4.86 (0.15)	-4.96 (0.16)	-1.43 (0.12)	-1.41 (0.13)	-5.08 (0.13)	0						
PS Eye	-1.49 (0.15)	-1.5 (0.15)	-1.46 (0.13)	-1.41 (0.15)	0.95 (0.12)	1.15 (0.11)	0					
PS Move	-1.45 (0.13)	-1.55 (0.15)	-1.49 (0.14)	-1.43 (0.14)	1.02 (0.09)	1.26 (0.09)	-1.63 (0.13)	0				
PS Wheel	-1.53 (0.12)	-1.48 (0.13)	-1.51 (0.13)	-1.47 (0.12)	1.06 (0.09)	1.2 (0.11)	-1.38 (0.14)	-1.39 (0.12)	0			
Wii	-4.79 (0.16)	-4.91 (0.17)	-1.43 (0.13)	-1.49 (0.13)	-4.83 (0.15)	-4.79 (0.16)	-1.38 (0.12)	-1.52 (0.13)	-1.45 (0.13)	0		
Wii U	-4.89 (0.17)	-4.87 (0.15)	-1.46 (0.14)	-1.31 (0.12)	-4.97 (0.13)	-4.81 (0.16)	-1.47 (0.12)	-1.46 (0.13)	-1.44 (0.16)	-4.99 (0.15)	0	
Wii Wheel	-1.59 (0.13)	-1.34 (0.12)	-1.47 (0.13)	-1.49 (0.12)	-1.5 (0.16)	-1.44 (0.14)	-1.48 (0.13)	-1.45 (0.14)	-1.53 (0.12)	1.02 (0.1)	1.11 (0.1)	0
Wii Motion	-1.45 (0.14)	-1.5 (0.15)	-1.58 (0.15)	-1.45 (0.13)	-1.46 (0.11)	-1.36 (0.12)	-1.45 (0.12)	-1.43 (0.12)	-1.4 (0.12)	1.18 (0.1)	1.25 (0.09)	-1.36 (0.12)

Table 19: Posterior Means of Individual Θ (std) - ALCMhetInv

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel	Wii Motion
Xbox One	-4.09	0											
	(0.12)												
Xbox Kinect	1.26	1.23	0										
	(0.11)	(0.12)											
Xbox Wheel	1.35	1.31	-1.56	0									
	(0.12)	(0.11)	(0.12)										
PS3	-4.04	-4.1	-1.51	-1.53	0								
	(0.12)	(0.12)	(0.14)	(0.11)									
PS4	-4.14	-4.19	-1.65	-1.59	-4.32	0							
	(0.13)	(0.12)	(0.11)	(0.11)	(0.11)								
PS Eye	-1.56	-1.63	-1.53	-1.59	1.16	1.28	0						
	(0.11)	(0.11)	(0.13)	(0.13)	(0.11)	(0.09)							
PS Move	-1.63	-1.57	-1.6	-1.62	1.16	1.37	-1.8	0					
	(0.12)	(0.13)	(0.13)	(0.13)	(0.1)	(0.11)	(0.13)						
PS Wheel	-1.63	-1.62	-1.61	-1.49	1.18	1.36	-1.52	-1.42	0				
	(0.11)	(0.14)	(0.13)	(0.12)	(0.11)	(0.09)	(0.12)	(0.11)					
Wii	-4	-4.09	-1.58	-1.57	-4.09	-4.04	-1.6	-1.58	-1.57	0			
	(0.12)	(0.12)	(0.12)	(0.13)	(0.12)	(0.1)	(0.13)	(0.12)	(0.13)				
Wii U	-4.07	-4.07	-1.59	-1.52	-4.15	-3.96	-1.57	-1.6	-1.54	-4.13	0		
	(0.12)	(0.12)	(0.12)	(0.11)	(0.12)	(0.11)	(0.14)	(0.12)	(0.12)	(0.12)			
Wii Wheel	-1.55	-1.56	-1.55	-1.59	-1.55	-1.59	-1.54	-1.58	-1.64	1.17	1.26	0	
	(0.13)	(0.11)	(0.11)	(0.16)	(0.1)	(0.11)	(0.12)	(0.12)	(0.13)	(0.1)	(0.08)		
Wii Motion	-1.56	-1.68	-1.55	-1.51	-1.62	-1.58	-1.63	-1.62	-1.51	1.33	1.35	-1.44	0
	(0.12)	(0.11)	(0.13)	(0.12)	(0.12)	(0.11)	(0.14)	(0.1)	(0.11)	(0.1)	(0.1)	(0.12)	
Inventory	-4.18	-4.15	-1.63	-1.7	-4.06	-4.03	-1.69	-1.65	-1.64	-4.06	-4.02	-1.65	-1.65
	(0.11)	(0.11)	(0.13)	(0.14)	(0.1)	(0.16)	(0.11)	(0.12)	(0.13)	(0.12)	(0.11)	(0.11)	(0.11)

Table 20: Posterior Means of Individual Θ (std) - ALCMhom

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox One	-2.74	0										
	(0.13)											
Xbox Kinect	1.57	0.87	0									
	(0.07)	(0.1)										
Xbox Wheel	2.32	1.84	-0.06	0								
	(0.05)	(0.05)	(0.13)									
PS3	-1.1	-0.97	-1.33	-0.25	0							
	(0.11)	(0.05)	(0.05)	(0.05)								
PS4	-1.08	-1.72	-0.97	-0.13	-3.03	0						
	(0.1)	(0.04)	(0.05)	(0.04)	(0.05)							
PS Eye	-0.7	-0.1	-0.28	0.22	0.43	1.41	0					
	(0.04)	(0.07)	(0.09)	(0.1)	(0.08)	(0.08)						
PS Move	-1.28	-0.73	-0.67	-0.29	0.89	1.73	-0.9	0				
	(0.06)	(0.16)	(0.1)	(0.06)	(0.06)	(0.09)	(0.07)					
PS Wheel	-0.47	0.16	-0.17	-0.09	1.19	1.69	0.84	0.44	0			
	(0.08)	(0.06)	(0.09)	(0.08)	(0.05)	(0.05)	(0.13)	(0.09)				
Wii	-0.94	-0.95	-0.08	-0.38	-1.37	-0.54	-0.99	-0.48	-0.39	0		
	(0.04)	(0.08)	(0.09)	(0.05)	(0.05)	(0.05)	(0.08)	(0.09)	(0.04)			
Wii U	-1.23	-0.85	-0.74	0.29	-1.66	-0.91	-0.62	-0.03	0.28	-2.18	0	
	(0.13)	(0.08)	(0.14)	(0.08)	(0.13)	(0.06)	(0.05)	(0.09)	(0.08)	(0.11)		
Wii Wheel	-0.63	-0.49	-0.32	-0.18	-0.4	-0.64	0.47	-0.73	-0.64	0.57	0.94	0
	(0.11)	(0.04)	(0.07)	(0.05)	(0.04)	(0.04)	(0.1)	(0.06)	(0.12)	(0.09)	(0.13)	
Wii Motion	-0.06	-0.53	-1.05	-0.11	-0.7	-0.43	-0.27	-0.81	-0.12	1.22	2.01	0.53
	(0.08)	(0.04)	(0.03)	(0.05)	(0.04)	(0.07)	(0.05)	(0.1)	(0.08)	(0.05)	(0.07)	(0.05)

Table 21: Correlation Matrix of Error Terms (std) - MvP

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox One	-0.03	1										
	(0.05)											
Xbox Kinect	0.51	0.38	1									
	(0.05)	(0.05)										
Xbox Wheel	0.45	0.47	0.24	1								
	(0.06)	(0.06)	(0.07)									
PS3	-0.17	-0.15	-0.18	-0.19	1							
	(0.07)	(0.08)	(0.07)	(0.06)								
PS4	-0.27	-0.34	-0.33	-0.13	-0.36	1						
	(0.06)	(0.08)	(0.04)	(0.06)	(0.05)							
PS Eye	-0.3	-0.29	-0.18	-0.23	0.09	0.4	1					
	(0.05)	(0.06)	(0.09)	(0.07)	(0.06)	(0.05)						
PS Move	-0.37	-0.21	-0.27	-0.12	0.21	0.53	-0.04	1				
	(0.05)	(0.09)	(0.05)	(0.08)	(0.07)	(0.04)	(0.05)					
PS Wheel	-0.24	-0.14	-0.32	-0.08	0.2	0.5	0.33	0.35	1			
	(0.06)	(0.08)	(0.08)	(0.08)	(0.06)	(0.05)	(0.04)	(0.04)				
Wii	-0.02	-0.05	-0.05	-0.25	-0.24	-0.12	-0.33	-0.19	-0.29	1		
	(0.05)	(0.04)	(0.07)	(0.06)	(0.06)	(0.08)	(0.06)	(0.08)	(0.07)			
Wii U	-0.09	-0.06	-0.15	-0.09	-0.36	0	-0.1	-0.14	-0.18	-0.15	1	
	(0.06)	(0.08)	(0.09)	(0.07)	(0.08)	(0.07)	(0.06)	(0.06)	(0.1)	(0.05)		
Wii Wheel	-0.11	0.06	-0.07	-0.1	-0.11	-0.27	-0.14	-0.28	-0.17	0.39	0.39	1
	(0.08)	(0.07)	(0.08)	(0.07)	(0.06)	(0.07)	(0.07)	(0.06)	(0.07)	(0.05)	(0.05)	
Wii Motion	-0.22	0.05	-0.26	-0.2	-0.27	-0.19	-0.3	-0.19	-0.13	0.48	0.51	0.46
	(0.06)	(0.07)	(0.08)	(0.08)	(0.06)	(0.05)	(0.05)	(0.05)	(0.07)	(0.04)	(0.04)	(0.06)

Table 22: Posterior Means of Cross-Price Effects (std) - IndepCPE

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel	Wii Motion
Xbox 360	0.00	0.00	-1.03	-0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(0.07)	(0.11)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Xbox One	0.00	0.00	0.00	-0.66	0.00	-0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(0.05)	(-)	(0.02)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Xbox Kinect	-0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(0.09)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Xbox Wheel	-0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(0.24)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
PS3	-0.41	0.00	0.00	0.00	0.00	-0.17	-0.41	-0.42	-0.30	0.00	0.00	0.00	0.00
	(0.15)	(-)	(-)	(-)	(-)	(0.03)	(0.08)	(0.09)	(0.10)	(-)	(-)	(-)	(-)
PS4	0.00	-0.45	0.00	0.00	-0.34	0.00	-0.62	-0.66	-0.77	0.00	-0.37	0.00	0.00
	(-)	(0.04)	(-)	(-)	(0.04)	(-)	(0.05)	(0.05)	(0.09)	(-)	(0.06)	(-)	(-)
PS Eye	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.17	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(0.16)	(-)	(-)	(-)	(-)	(-)
PS Move	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
PS Wheel	0.00	0.00	0.00	0.00	-0.55	0.00	0.00	-0.31	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(0.19)	(-)	(-)	(0.19)	(-)	(-)	(-)	(-)	(-)
Wii	-0.45	0.00	0.00	0.00	-0.26	0.00	0.00	0.00	0.00	0.00	-0.50	-0.35	-0.32
	(0.07)	(-)	(-)	(-)	(0.08)	(-)	(-)	(-)	(-)	(-)	(0.10)	(0.06)	(0.07)
Wii U	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.57	0.00	-0.62	-0.53
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(0.03)	(-)	(0.04)	(0.04)
Wii Wheel	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Wii Motion	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)

Table 23: Heterogeneity of Individual Θ (std) - ALCMhet

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox One	2.91 (0.09)	0										
Xbox Kinect	2.26 (0.09)	2.18 (0.09)	0									
Xbox Wheel	2.31 (0.08)	2.24 (0.08)	2.88 (0.11)	0								
PS3	2.95 (0.08)	2.88 (0.08)	2.85 (0.09)	2.9 (0.1)	0							
PS4	2.92 (0.09)	2.88 (0.09)	2.8 (0.11)	2.84 (0.09)	2.82 (0.08)	0						
PS Eye	2.84 (0.11)	2.88 (0.12)	2.87 (0.1)	2.8 (0.12)	2.08 (0.1)	2.24 (0.09)	0					
PS Move	2.85 (0.1)	2.87 (0.11)	2.85 (0.1)	2.86 (0.1)	2.15 (0.09)	2.33 (0.08)	2.93 (0.09)	0				
PS Wheel	2.89 (0.09)	2.84 (0.1)	2.86 (0.1)	2.91 (0.09)	2.18 (0.09)	2.29 (0.09)	2.92 (0.1)	2.87 (0.1)	0			
Wii	2.96 (0.08)	2.9 (0.1)	2.87 (0.11)	2.89 (0.1)	2.94 (0.07)	2.96 (0.08)	2.81 (0.11)	2.86 (0.09)	2.88 (0.11)	0		
Wii U	2.91 (0.08)	2.92 (0.08)	2.88 (0.1)	2.79 (0.11)	2.87 (0.08)	2.95 (0.08)	2.85 (0.1)	2.88 (0.1)	2.86 (0.1)	2.86 (0.08)	0	
Wii Wheel	2.91 (0.09)	2.8 (0.1)	2.88 (0.1)	2.89 (0.09)	2.87 (0.11)	2.83 (0.1)	2.91 (0.1)	2.83 (0.12)	2.9 (0.09)	2.15 (0.09)	2.23 (0.09)	0
Wii Motion	2.85 (0.1)	2.89 (0.1)	2.91 (0.1)	2.83 (0.1)	2.85 (0.1)	2.79 (0.1)	2.84 (0.1)	2.82 (0.09)	2.86 (0.09)	2.27 (0.08)	2.33 (0.07)	2.88 (0.1)

Table 24: Heterogeneity of Individual Θ (std) - ALCMhetInv

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel	Wii Motion
Xbox One	2.47 (0.06)	0											
Xbox Kinect	2.17 (0.08)	2.14 (0.09)	0										
Xbox Wheel	2.21 (0.08)	2.18 (0.08)	2.67 (0.07)	0									
PS3	2.5 (0.06)	2.47 (0.07)	2.66 (0.09)	2.64 (0.08)	0								
PS4	2.45 (0.08)	2.44 (0.07)	2.68 (0.06)	2.7 (0.07)	2.38 (0.07)	0							
PS Eye	2.65 (0.07)	2.66 (0.07)	2.66 (0.08)	2.64 (0.08)	2.08 (0.08)	2.15 (0.07)	0						
PS Move	2.66 (0.07)	2.64 (0.07)	2.67 (0.08)	2.7 (0.07)	2.09 (0.08)	2.22 (0.08)	2.75 (0.07)	0					
PS Wheel	2.69 (0.07)	2.65 (0.08)	2.66 (0.08)	2.7 (0.07)	2.11 (0.08)	2.2 (0.07)	2.79 (0.08)	2.69 (0.08)	0				
Wii	2.52 (0.06)	2.48 (0.07)	2.7 (0.08)	2.67 (0.07)	2.48 (0.06)	2.51 (0.05)	2.69 (0.07)	2.65 (0.07)	2.65 (0.07)	0			
Wii U	2.48 (0.06)	2.5 (0.07)	2.69 (0.07)	2.65 (0.08)	2.45 (0.07)	2.53 (0.06)	2.71 (0.08)	2.7 (0.07)	2.66 (0.07)	2.46 (0.06)	0		
Wii Wheel	2.65 (0.08)	2.69 (0.07)	2.69 (0.07)	2.65 (0.09)	2.68 (0.08)	2.67 (0.07)	2.64 (0.08)	2.64 (0.07)	2.69 (0.09)	2.1 (0.08)	2.16 (0.07)	0	
Wii Motion	2.67 (0.07)	2.76 (0.06)	2.62 (0.08)	2.63 (0.07)	2.69 (0.07)	2.7 (0.07)	2.67 (0.07)	2.69 (0.07)	2.67 (0.07)	2.19 (0.07)	2.21 (0.07)	2.79 (0.08)	0
Inventory	2.41 (0.07)	2.44 (0.07)	2.71 (0.07)	2.69 (0.07)	2.48 (0.06)	2.49 (0.07)	2.7 (0.07)	2.66 (0.07)	2.71 (0.07)	2.48 (0.07)	2.5 (0.06)	2.7 (0.08)	2.66 (0.07)

Table 25: Heterogeneity in Cross-Price Effects (std) - IndepCPE

	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel	Wii Motion
Xbox 360	0.00	0.00	0.81	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(0.04)	(0.05)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Xbox One	0.00	0.00	0.00	0.57	0.00	0.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(0.03)	(-)	(0.02)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Xbox Kinect	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(0.05)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Xbox Wheel	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(0.12)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
PS3	0.78	0.00	0.00	0.00	0.00	0.65	0.67	0.81	0.69	0.00	0.00	0.00	0.00
	(0.07)	(-)	(-)	(-)	(-)	(0.04)	(0.04)	(0.06)	(0.05)	(-)	(-)	(-)	(-)
PS4	0.00	0.64	0.00	0.00	0.59	0.00	0.61	0.68	0.68	0.00	0.61	0.00	0.00
	(-)	(0.02)	(-)	(-)	(0.03)	(-)	(0.04)	(0.04)	(0.06)	(-)	(0.05)	(-)	(-)
PS Eye	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.19	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(0.12)	(-)	(-)	(-)	(-)	(-)
PS Move	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
PS Wheel	0.00	0.00	0.00	0.00	1.36	0.00	0.00	1.34	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(0.14)	(-)	(-)	(0.14)	(-)	(-)	(-)	(-)	(-)
Wii	0.75	0.00	0.00	0.00	0.73	0.00	0.00	0.00	0.00	0.00	0.74	0.64	0.61
	(0.04)	(-)	(-)	(-)	(0.07)	(-)	(-)	(-)	(-)	(-)	(0.06)	(0.04)	(0.04)
Wii U	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.00	0.62	0.60
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(0.03)	(-)	(0.03)	(0.03)
Wii Wheel	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
Wii Motion	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)

Table 26: Correlation Matrix of Individual Preferences (std) - ALCMhet

	Price	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox 360	-0.42	1											
	(0.05)												
Xbox One	-0.42	0.45	1										
	(0.06)	(0.07)											
Xbox Kinect	-0.17	0.19	0.24	1									
	(0.08)	(0.11)	(0.11)										
Xbox Wheel	-0.21	0.17	0.04	0.65	1								
	(0.06)	(0.09)	(0.1)	(0.09)									
PS3	-0.39	0.87	0.32	-0.09	0.04	1							
	(0.06)	(0.02)	(0.1)	(0.12)	(0.09)								
PS4	-0.38	0.4	0.78	-0.06	-0.08	0.47	1						
	(0.05)	(0.07)	(0.04)	(0.12)	(0.1)	(0.06)							
PS Eye	-0.13	0.18	-0.08	0.48	0.57	0.19	0.11	1					
	(0.07)	(0.09)	(0.09)	(0.11)	(0.08)	(0.11)	(0.09)						
PS Move	-0.28	0.2	0.36	0.54	0.54	0.2	0.48	0.7	1				
	(0.06)	(0.08)	(0.08)	(0.09)	(0.08)	(0.08)	(0.06)	(0.06)					
PS Wheel	-0.15	0	0.03	0.35	0.71	0.06	0.22	0.61	0.69	1			
	(0.06)	(0.09)	(0.1)	(0.1)	(0.08)	(0.09)	(0.09)	(0.07)	(0.06)				
Wii	-0.38	0.63	0.1	0.08	0.24	0.59	-0.02	0.09	-0.06	-0.05	1		
	(0.06)	(0.06)	(0.12)	(0.1)	(0.09)	(0.06)	(0.08)	(0.1)	(0.1)	(0.1)			
Wii U	-0.48	0.26	0.63	0.14	0.19	0.24	0.52	0.05	0.27	0.14	0.5	1	
	(0.05)	(0.08)	(0.06)	(0.1)	(0.1)	(0.09)	(0.07)	(0.09)	(0.12)	(0.1)	(0.09)		
Wii Wheel	-0.27	0.22	-0.01	0.54	0.79	0.1	-0.23	0.33	0.3	0.49	0.48	0.31	1
	(0.06)	(0.08)	(0.1)	(0.11)	(0.06)	(0.08)	(0.08)	(0.09)	(0.08)	(0.09)	(0.06)	(0.09)	
Wii Motion	-0.31	0.19	0.03	0.53	0.64	0.06	-0.24	0.28	0.26	0.28	0.59	0.46	0.83
	(0.06)	(0.08)	(0.09)	(0.08)	(0.07)	(0.09)	(0.08)	(0.11)	(0.11)	(0.12)	(0.06)	(0.07)	(0.04)

Table 27: Correlation Matrix of Individual Preferences (std) - ALCMhetInv

	Price	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox 360	-0.53	1											
	(0.05)												
Xbox One	-0.48	0.57	1										
	(0.05)	(0.08)											
Xbox Kinect	-0.26	0.28	0.25	1									
	(0.08)	(0.12)	(0.12)										
Xbox Wheel	-0.25	0.32	0.03	0.73	1								
	(0.06)	(0.1)	(0.1)	(0.06)									
PS3	-0.51	0.92	0.57	0.14	0.21	1							
	(0.05)	(0.02)	(0.07)	(0.12)	(0.09)								
PS4	-0.43	0.56	0.91	0.15	-0.06	0.63	1						
	(0.06)	(0.09)	(0.02)	(0.12)	(0.1)	(0.07)							
PS Eye	-0.18	0.27	0.07	0.64	0.64	0.27	0.15	1					
	(0.07)	(0.08)	(0.1)	(0.09)	(0.07)	(0.08)	(0.09)						
PS Move	-0.24	0.26	0.41	0.68	0.59	0.27	0.46	0.73	1				
	(0.07)	(0.09)	(0.08)	(0.08)	(0.08)	(0.1)	(0.08)	(0.06)					
PS Wheel	-0.18	0.18	0.16	0.6	0.71	0.21	0.21	0.67	0.8	1			
	(0.07)	(0.08)	(0.09)	(0.09)	(0.06)	(0.09)	(0.08)	(0.07)	(0.05)				
Wii	-0.46	0.72	0.23	0.18	0.32	0.65	0.15	0.17	-0.09	-0.01	1		
	(0.06)	(0.05)	(0.1)	(0.11)	(0.09)	(0.06)	(0.13)	(0.1)	(0.11)	(0.09)			
Wii U	-0.55	0.59	0.76	0.42	0.32	0.51	0.61	0.19	0.31	0.2	0.62	1	
	(0.05)	(0.08)	(0.04)	(0.1)	(0.09)	(0.07)	(0.06)	(0.11)	(0.09)	(0.08)	(0.09)		
Wii Wheel	-0.29	0.31	-0.01	0.55	0.78	0.19	-0.2	0.38	0.27	0.51	0.51	0.41	1
	(0.06)	(0.1)	(0.09)	(0.1)	(0.05)	(0.1)	(0.08)	(0.09)	(0.09)	(0.08)	(0.07)	(0.08)	
Wii Motion	-0.34	0.37	0.04	0.53	0.68	0.22	-0.14	0.35	0.16	0.3	0.65	0.52	0.82
	(0.06)	(0.08)	(0.1)	(0.1)	(0.06)	(0.09)	(0.09)	(0.09)	(0.1)	(0.1)	(0.06)	(0.07)	(0.04)

Table 28: Correlation Matrix of Individual Preferences (std) - ALCMhom

	Price	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox 360	-0.4	1											
	(0.05)												
Xbox One	-0.37	0.51	1										
	(0.06)	(0.08)											
Xbox Kinect	-0.15	0.29	0.34	1									
	(0.06)	(0.08)	(0.09)										
Xbox Wheel	-0.11	0.26	0.08	0.6	1								
	(0.07)	(0.08)	(0.1)	(0.07)									
PS3	-0.39	0.81	0.35	0.02	0.07	1							
	(0.05)	(0.03)	(0.09)	(0.09)	(0.08)								
PS4	-0.35	0.3	0.73	-0.04	-0.2	0.45	1						
	(0.06)	(0.08)	(0.05)	(0.09)	(0.08)	(0.07)							
PS Eye	-0.09	0.16	-0.02	0.45	0.28	0.28	0.08	1					
	(0.06)	(0.09)	(0.1)	(0.08)	(0.08)	(0.08)	(0.08)						
PS Move	-0.18	0.19	0.35	0.62	0.46	0.23	0.34	0.56	1				
	(0.06)	(0.09)	(0.08)	(0.07)	(0.08)	(0.08)	(0.07)	(0.06)					
PS Wheel	-0.12	0.07	0.04	0.19	0.64	0.17	0.22	0.26	0.49	1			
	(0.06)	(0.09)	(0.09)	(0.09)	(0.06)	(0.08)	(0.07)	(0.08)	(0.07)				
Wii	-0.31	0.54	-0.11	0.14	0.23	0.52	-0.16	0.24	-0.09	0.04	1		
	(0.06)	(0.07)	(0.1)	(0.09)	(0.09)	(0.07)	(0.09)	(0.09)	(0.09)	(0.09)			
Wii U	-0.41	0.29	0.49	0.23	0.09	0.21	0.36	0.11	0.07	0.03	0.46	1	
	(0.06)	(0.1)	(0.08)	(0.1)	(0.1)	(0.1)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)		
Wii Wheel	-0.18	0.22	0	0.52	0.78	0.02	-0.3	0.16	0.26	0.47	0.44	0.37	1
	(0.06)	(0.08)	(0.1)	(0.08)	(0.05)	(0.08)	(0.08)	(0.08)	(0.08)	(0.07)	(0.07)	(0.08)	
Wii Motion	-0.24	0.21	-0.04	0.54	0.5	0.08	-0.22	0.28	0.34	0.22	0.6	0.42	0.67
	(0.06)	(0.09)	(0.09)	(0.08)	(0.07)	(0.08)	(0.08)	(0.09)	(0.08)	(0.08)	(0.05)	(0.08)	(0.05)

Table 29: Correlation Matrix of Individual Preferences (std) - Indep Model

	Price	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox 360	-0.34	1											
	(0.06)												
Xbox One	-0.31	0.38	1										
	(0.08)	(0.1)											
Xbox Kinect	-0.08	0.53	0.41	1									
	(0.07)	(0.07)	(0.1)										
Xbox Wheel	-0.02	0.48	0.2	0.67	1								
	(0.06)	(0.07)	(0.09)	(0.06)									
PS3	-0.33	0.74	0.12	0.1	0.17	1							
	(0.06)	(0.04)	(0.1)	(0.09)	(0.08)								
PS4	-0.3	0.05	0.56	-0.15	-0.25	0.22	1						
	(0.06)	(0.09)	(0.08)	(0.1)	(0.08)	(0.08)							
PS Eye	-0.06	0.1	0.04	0.24	0.2	0.31	0.36	1					
	(0.06)	(0.08)	(0.1)	(0.09)	(0.08)	(0.08)	(0.06)						
PS Move	-0.14	0.11	0.33	0.26	0.21	0.28	0.6	0.6	1				
	(0.06)	(0.08)	(0.08)	(0.09)	(0.08)	(0.08)	(0.05)	(0.06)					
PS Wheel	-0.06	0.06	0.07	0.03	0.41	0.24	0.46	0.52	0.65	1			
	(0.06)	(0.08)	(0.08)	(0.09)	(0.07)	(0.08)	(0.06)	(0.06)	(0.05)				
Wii	-0.3	0.49	-0.22	0.12	0.17	0.45	-0.27	0.01	-0.25	-0.13	1		
	(0.07)	(0.07)	(0.13)	(0.1)	(0.08)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)			
Wii U	-0.38	0.13	0.4	0.12	0.09	0.02	0.2	0.03	0.03	-0.02	0.41	1	
	(0.07)	(0.1)	(0.1)	(0.1)	(0.08)	(0.1)	(0.08)	(0.08)	(0.08)	(0.07)	(0.08)		
Wii Wheel	-0.12	0.25	-0.06	0.39	0.64	0.03	-0.42	0.01	-0.1	0.15	0.53	0.53	1
	(0.07)	(0.08)	(0.1)	(0.08)	(0.05)	(0.07)	(0.07)	(0.08)	(0.08)	(0.07)	(0.06)	(0.07)	
Wii Motion	-0.19	0.18	-0.13	0.31	0.37	0.01	-0.33	0.02	-0.05	-0.02	0.69	0.61	0.77
	(0.06)	(0.07)	(0.09)	(0.08)	(0.07)	(0.08)	(0.07)	(0.08)	(0.08)	(0.07)	(0.04)	(0.05)	(0.04)

Table 30: Correlation Matrix of Individual Preferences (std) - MvP

	Price	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox 360	-0.44	1											
	(0.06)												
Xbox One	-0.52	0.36	1										
	(0.08)	(0.08)											
Xbox Kinect	-0.02	0.42	0.31	1									
	(0.09)	(0.08)	(0.09)										
Xbox Wheel	0.04	0.41	0.14	0.55	1								
	(0.07)	(0.07)	(0.09)	(0.07)									
PS3	-0.49	0.67	0.19	0.08	0.14	1							
	(0.06)	(0.05)	(0.09)	(0.08)	(0.07)								
PS4	-0.49	0.1	0.5	-0.16	-0.25	0.24	1						
	(0.06)	(0.08)	(0.07)	(0.09)	(0.08)	(0.07)							
PS Eye	-0.01	0.09	-0.02	0.22	0.2	0.23	0.28	1					
	(0.08)	(0.08)	(0.09)	(0.08)	(0.09)	(0.07)	(0.07)						
PS Move	-0.16	0.1	0.21	0.23	0.17	0.23	0.51	0.52	1				
	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.07)	(0.05)	(0.05)					
PS Wheel	0.01	0.04	-0.03	0.06	0.35	0.16	0.37	0.44	0.56	1			
	(0.07)	(0.08)	(0.09)	(0.1)	(0.08)	(0.07)	(0.07)	(0.07)	(0.06)				
Wii	-0.42	0.44	0	0.08	0.15	0.4	-0.18	-0.05	-0.23	-0.18	1		
	(0.05)	(0.07)	(0.09)	(0.1)	(0.09)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)			
Wii U	-0.51	0.13	0.39	0.07	0.07	0.06	0.23	0.02	0.02	-0.05	0.38	1	
	(0.07)	(0.09)	(0.09)	(0.09)	(0.08)	(0.09)	(0.08)	(0.08)	(0.08)	(0.08)	(0.07)		
Wii Wheel	-0.14	0.21	-0.02	0.29	0.54	0.04	-0.36	0.01	-0.08	0.1	0.46	0.43	1
	(0.07)	(0.08)	(0.09)	(0.08)	(0.06)	(0.08)	(0.07)	(0.08)	(0.08)	(0.07)	(0.07)	(0.07)	
Wii Motion	-0.18	0.16	-0.06	0.23	0.35	0.01	-0.32	-0.01	-0.08	-0.06	0.61	0.51	0.69
	(0.07)	(0.07)	(0.09)	(0.09)	(0.08)	(0.08)	(0.07)	(0.07)	(0.07)	(0.08)	(0.05)	(0.06)	(0.04)

Table 31: Correlation Matrix of Individual Preferences (std) - IndepCPE

	Price	Xbox 360	Xbox One	Xbox Kinect	Xbox Wheel	PS3	PS4	PS Eye	PS Move	PS Wheel	Wii	Wii U	Wii Wheel
Xbox 360	-0.6	1											
	(0.05)												
Xbox One	-0.63	0.92	1										
	(0.04)	(0.03)											
Xbox Kinect	-0.61	0.9	0.93	1									
	(0.05)	(0.04)	(0.03)										
Xbox Wheel	-0.58	0.88	0.9	0.88	1								
	(0.05)	(0.06)	(0.05)	(0.05)									
PS3	-0.63	0.93	0.96	0.93	0.91	1							
	(0.04)	(0.03)	(0.02)	(0.03)	(0.05)								
PS4	-0.63	0.92	0.95	0.91	0.9	0.96	1						
	(0.04)	(0.05)	(0.03)	(0.06)	(0.05)	(0.02)							
PS Eye	-0.61	0.89	0.93	0.9	0.88	0.94	0.94	1					
	(0.04)	(0.05)	(0.03)	(0.04)	(0.05)	(0.02)	(0.02)						
PS Move	-0.63	0.92	0.95	0.92	0.91	0.96	0.97	0.94	1				
	(0.04)	(0.04)	(0.03)	(0.05)	(0.05)	(0.02)	(0.01)	(0.02)					
PS Wheel	-0.62	0.93	0.95	0.92	0.91	0.96	0.97	0.94	0.97	1			
	(0.04)	(0.04)	(0.03)	(0.04)	(0.05)	(0.02)	(0.01)	(0.02)	(0.01)				
Wii	-0.63	0.92	0.96	0.93	0.9	0.96	0.96	0.93	0.95	0.95	1		
	(0.04)	(0.04)	(0.02)	(0.03)	(0.05)	(0.02)	(0.04)	(0.03)	(0.03)	(0.03)			
Wii U	-0.63	0.93	0.96	0.93	0.9	0.96	0.96	0.93	0.96	0.96	0.96	1	
	(0.04)	(0.03)	(0.02)	(0.03)	(0.05)	(0.01)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)		
Wii Wheel	-0.61	0.9	0.92	0.89	0.89	0.93	0.95	0.91	0.94	0.94	0.92	0.93	1
	(0.05)	(0.06)	(0.04)	(0.06)	(0.06)	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.05)	(0.04)	
Wii Motion	-0.61	0.91	0.94	0.91	0.89	0.94	0.95	0.92	0.95	0.95	0.94	0.95	0.92
	(0.04)	(0.04)	(0.02)	(0.04)	(0.05)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)